Prince Edward Island
CANADA

Education and Early Years

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

MAT521A

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Table of Contents

Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for Grades 10-12 Mathematics* (2008) has been adopted as the basis for a revised high school mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential Graduation Learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to:

- respond with critical awareness to various forms of the arts and be able to express themselves through the arts.
- assess social, cultural, economic, and environmental interdependence in a local and global context.
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively.
- continue to learn and to pursue an active, healthy lifestyle.
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts.
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems.
- communicate and reason mathematically.
- appreciate and value mathematics.
- make connections between mathematics and its applications.
- commit themselves to lifelong learning.
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

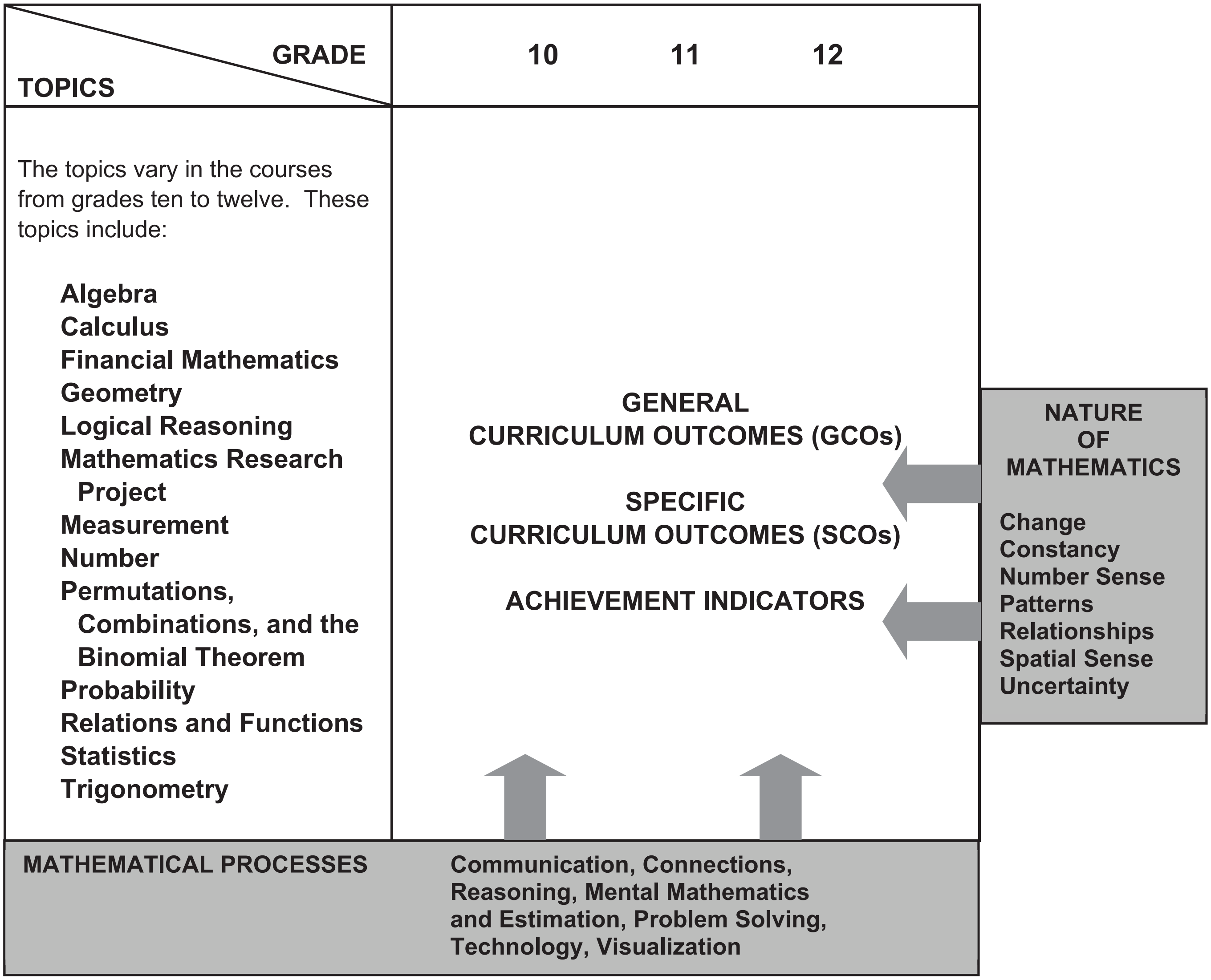
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art.
- exhibit a positive attitude toward mathematics.
- engage and persevere in mathematical tasks and projects.
- contribute to mathematical discussions.
- take risks in performing mathematical tasks.
- exhibit curiosity.

➢ Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practice mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for 10-12 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



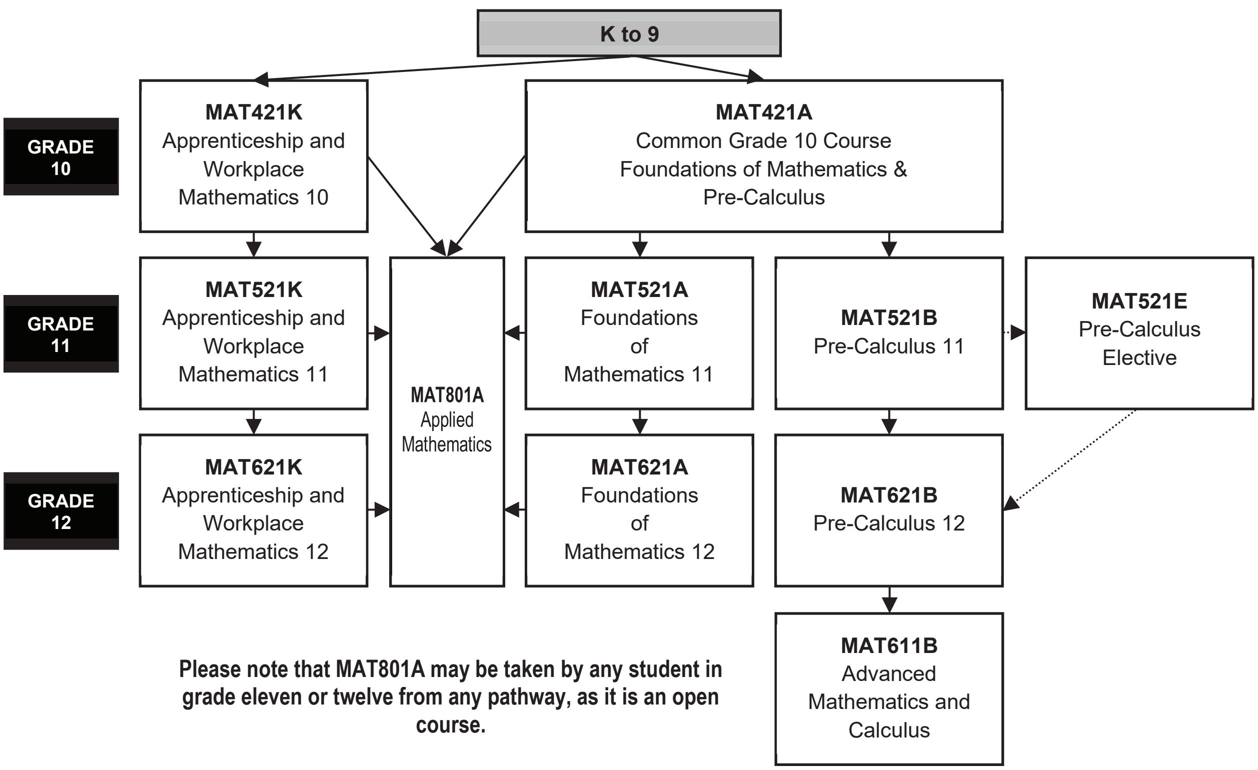
The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into a number of topics, as described above. These topics are not intended to be discrete units of instruction. The integration of outcomes across topics makes mathematical experiences meaningful. Students should make the connections among concepts both within and across topics. Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.

- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Pathways and Topics

The Prince Edward Island 10-12 mathematics curriculum includes pathways with corresponding topics rather than strands, which are found in the Prince Edward Island K-9 mathematics curriculum. Three pathways are available: **Apprenticeship and Workplace Mathematics**, **Foundations of Mathematics**, and **Pre-Calculus**. A common grade ten course (Foundations of Mathematics and Pre-Calculus, Grade 10) is the starting point for the Foundations of Mathematics pathway and the Pre-Calculus pathway. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings. These pathways are illustrated in the diagram below:



The goals of all three pathways are to provide the prerequisite knowledge, skills, understandings, and attitudes for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study or for direct entry into the work force. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Apprenticeship and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics, and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, algebra and number, logical reasoning, relations and functions, statistics, probability, and a mathematics research project.

Pre-Calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, combinatorics, and introductory calculus.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to:

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

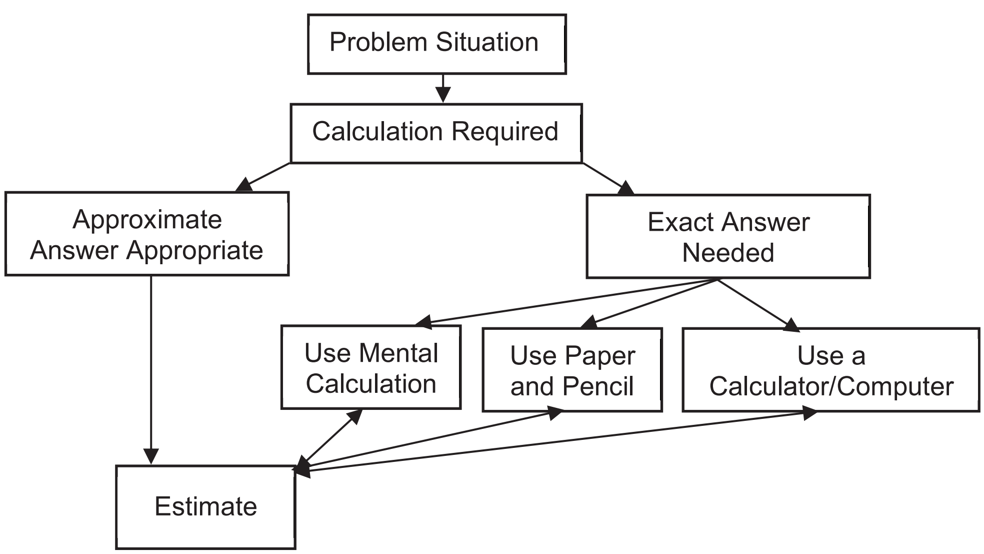
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?" or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns.
- organize and display data.
- extrapolate and interpolate.
- assist with calculation procedures as part of solving problems.
- decrease the time spent on computations when other mathematical learning is the focus.
- reinforce the learning of basic facts and test properties.
- develop personal procedures for mathematical operations.
- create geometric displays.
- simulate situations.

- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4.
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.

- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all topics in mathematics and it is important that connections are made among topics. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects and identify relationships to mathematical topics. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education" (p.60). The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate “communicating to learn mathematics and learning to communicate mathematically” (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics.
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics.
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors.
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage.
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents.
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

➢ Inquiry-Based Learning and Project Based Learning

Inquiry-based learning (IBL) allows students to explore, investigate, and construct new meaning from prior knowledge and from new information that is retrieved from other sources. It is not linear in nature, but promotes a continual looping back and forth throughout the process as students gather and process new information, redirect their inquiries, and continue through the process. Mathematical inquiry will require students to practise and refine their critical and creative-thinking skills. The terms *inquiry* and *research* are often used interchangeably within an educational context. While research often becomes the end-result of an inquiry process, it is the process itself that should be emphasized within an educational context. More information regarding the development of a mathematics research project is included in the appendix at the end of this document.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, teaching has been effective, or how best to address student learning needs. The quality of assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

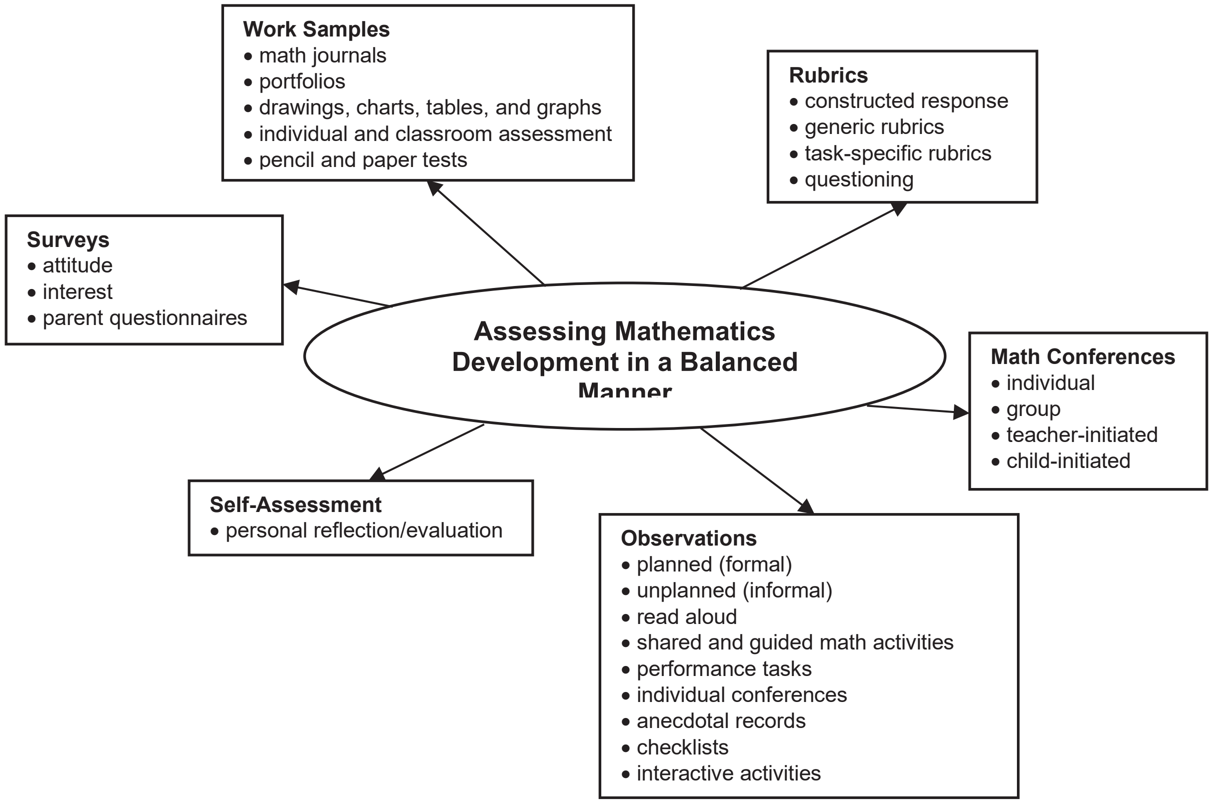
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as:

- providing feedback to improve student learning.
- determining if curriculum outcomes have been achieved.
- certifying that students have achieved certain levels of performance.
- setting goals for future student learning.
- communicating with parents about their children's learning.
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment.
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including:

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used:

- to engage students in their own learning and self-assessment.
- to help students understand what is important in the mathematical concepts and particular tasks they encounter.
- to develop effective habits of metacognition and self-coaching.
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used:

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups.
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed.
- to provide feedback to students about how they are doing and how they might improve.
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used:

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units.
- to facilitate reporting.

- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires:

- developing clear criteria and guidelines for assigning marks or grades to student work.
- synthesizing information from multiple sources.
- weighing and balancing all available information.
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, phone calls, and electronic methods.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that:

- the best interests of the student are paramount.

- assessment informs teaching and promotes learning.
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes.
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he or she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island high school mathematics curriculum are organized into a number of topics across the grades from ten to twelve. Each topic has associated with it a general curriculum outcome (GCO). They are overarching statements about what students are expected to learn in each topic from grades ten to twelve.

Topic	General Curriculum Outcome (GCO)
Algebra (A)	Develop algebraic reasoning.
Algebra and Number (AN)	Develop algebraic reasoning and number sense.
Calculus (C)	Develop introductory calculus reasoning.
Financial Mathematics (FM)	Develop number sense in financial applications.
Geometry (G)	Develop spatial sense.
Logical Reasoning (LR)	Develop logical reasoning.
Mathematics Research Project (MRP)	Develop an appreciation of the role of mathematics in society.
Measurement (M)	Develop spatial sense and proportional reasoning. (***Foundations of Mathematics and Pre-Calculus***)
	Develop spatial sense through direct and indirect measurement. (***Apprenticeship and Workplace Mathematics***)
Number (N)	Develop number sense and critical thinking skills.
Permutations, Combinations and Binomial Theorem (PC)	Develop algebraic and numeric reasoning that involves combinatorics.
Probability (P)	Develop critical thinking skills related to uncertainty.
Relations and Functions (RF)	Develop algebraic and graphical reasoning through the study of relations.
Statistics (S)	Develop statistical reasoning.
Trigonometry (T)	Develop trigonometric reasoning.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific curriculum outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding topic and general curriculum outcome.
- the scope and sequence of the specific curriculum outcome(s) from grades ten to twelve which correspond to this SCO.
- the specific curriculum outcome, with a list of achievement indicators.

- a list of the sections in *Foundations of Mathematics 11* which address the SCO, with specific achievement indicators highlighted in brackets.
- an elaboration for the SCO.

MEASUREMENT

SPECIFIC CURRICULUM OUTCOMES

M1 – Solve problems that involve the application of rates.

M2 – Solve problems that involve scale diagrams, using proportional reasoning.

M3 – Demonstrate an understanding of the relationships between scale factors and areas of similar 2-D shapes, and the application of scale factor on 3-D objects.

MAT521A – Topic: Measurement (M)

GCO: Develop spatial sense and proportional reasoning.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
M2 Apply proportional reasoning to solve problems involving conversions between and within SI and imperial units of measure.	**M1** Solve problems that involve the application of rates.	

SCO: **M1 – Solve problems that involve the application of rates.** [CN, PS, R]

Students who have achieved this outcome should be able to:

A. Interpret rates in a given context, such as the arts, commerce, the environment, medicine or recreation.

B. Solve a rate problem that requires the isolation of a variable.

C. Determine and compare rates and unit rates.

D. Make and justify a decision, using rates.

E. Explain, using examples, the relationship between the slope of a graph and a rate.

F. Describe a context for a given rate or unit rate.

G. Identify and explain factors that influence a rate in a given context.

H. Solve a contextual problem that involves rates or unit rates.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

8.1 (A C D E F H)

8.2 (B G H)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: M1 – Solve problems that involve the application of rates. [CN, PS, R]

Elaboration

Rates can be represented in a variety of ways. The representation that is chosen depends on the purpose. In any case, the same units must be used to compare rates. Consider the following, where:

- both rates are written with the second terms equal. E.g. 18 km : 3 hours as compared to 6 km : 3 hours.
- Both rates are unit rates. E. g. 6 km : 1 hour compared to 2 km : 1 hour.

When comparing rates, it is helpful to round values. This will enable students to do mental math and express each rate as an approximate unit rate.

In a graph that shows the relationship between two quantities, the slope of a line segment represents the average rate of change for these quantities. The slope of a line segment that represents a rate of change is a unit rate.

When solving a rate problem that involves an unknown, the problem can be solved using a variety of strategies. A common method is to solve the problem by writing an equation that involves an equivalent pair of ratios, called a proportion. To be equivalent ratios, the units in the numerators of the two ratios must be the same, and the units in the denominators of the two ratios must be the same. This should help students write proportions correctly when solving a rate problem.

As well, a multiplication strategy can be used to solve many rate problems, such as problems that require conversions between units. Including the units with each term in the product and using unit elimination will help verify that the product is correct.

Finally, when the rate of change is constant, writing a linear function to represent the situation may be helpful when solving problems.

MAT521A – Topic: Measurement (M)

GCO: Develop spatial sense and proportional reasoning.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
M2 Apply proportional reasoning to solve problems involving conversions between and within SI and imperial units of measure.	**M2** Solve problems that involve scale diagrams, using proportional reasoning.	

SCO: **M2 – Solve problems that involve scale diagrams, using proportional reasoning.** **[CN, PS, R, V]**

Students who have achieved this outcome should be able to:

A. Explain, using examples, how scale diagrams are used to model a 2-D shape or a 3-D object.

B. Determine, using proportional reasoning, the scale factor, given one dimension of a 2-D shape or a 3-D object and its representation.

C. Determine, using proportional reasoning, an unknown dimension of a 2-D shape or a 3-D object, given a scale diagram or a model.

D. Solve a contextual problem that involves scale diagrams.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

8.3 (A B C D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: M2 – Solve problems that involve scale diagrams, using proportional reasoning. [CN, PS, R, V]

Elaboration

Scale diagrams can be used to represent 2-D shapes. To create a scale diagram, it is important to determine an appropriate scale to use. This depends on the dimensions of the original shape and the size of the diagram that is required. The scale represents the ratio of a distance measurement of a shape to the corresponding distance measurement of a similar shape, where both measurements are expressed using the same units.

A scale factor is a number created from the ratio of any two corresponding measurements of two similar shapes or objects. Any linear dimension of a shape can be multiplied by the scale factor to calculate the corresponding linear dimension of a similar shape. When determining the scale factor, *k*, used for a scale diagram, the measurement from the original shape is placed in the denominator:

$$k = \frac{\text{Diagram measurement}}{\text{Actual measurement}}$$

When a scale factor is between 0 and 1, the new shape is a reduction of the original shape, and when a scale factor is greater than 1, the new shape is an enlargement of the original shape.

MAT521A – Topic: Measurement (M)

GCO: Develop spatial sense and proportional reasoning.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
M3 Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including: • right cones; • right cylinders; • right prisms; • right pyramids; • spheres.	**M3** Demonstrate an understanding of the relationships between scale factors and areas of similar 2-D shapes, and the application of scale factor on 3-D objects.	

SCO: **M3 – Demonstrate an understanding of the relationships between scale factors and areas of similar 2-D shapes, and the application of scale factor on 3-D objects.** **[C, CN, PS, R, V]**

Students who have achieved this outcome should be able to:

A. Determine the area of a 2-D shape, given the scale diagram, and justify the reasonableness of the result.

B. Explain, using examples, the effect of a change in the scale factor on the area of a 2-D shape.

C. Explain, using examples, the relationship between scale factor and area of a 2-D shape.

D. Determine the dimensions of a similar 3-D object using scale factor.

E. Solve a contextual problem that involves the relationships between scale factors and areas for 2 – D shapes.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

8.4 (A B C D E)

8.5 (C D E)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: M3 – Demonstrate an understanding of the relationships between scale factors and areas of similar 2-D shapes, and the application of scale factor on 3-D objects. [C, CN, PS, R, V]

Elaboration

If two 2-D shapes are similar and their dimensions are related by a scale factor of *k*, then the relationship between the area of the similar shape and the area of the original shape can be expressed as:

$$\text{Area of similar 2-D shape} = k^2\left(\text{Area of original shape}\right)$$

If the area of a similar 2-D shape and the area of the original shape are known, then the scale factor, *k*, can be determined using the formula:

$$k^2 = \frac{\text{Area of similar 2-D shape}}{\text{Area of original shape}}$$

Two 3-D objects that are similar have dimensions that are proportional. The scale factor is the ratio of a linear measurement of an object to the corresponding linear measurement in a similar object, where both measurements are expressed in the same units. To create a scale model or diagram, determine an appropriate scale to use based on the dimensions of the original shape and the size of the model or diagram that is required.

GEOMETRY

SPECIFIC CURRICULUM OUTCOMES

G1 – Derive proofs that involve the properties of angles and triangles.

G2 – Solve problems that involve the properties of angles and triangles.

G3 – Solve problems that involve the cosine law and the sine law.

MAT521A – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.	**G1** Derive proofs that involve the properties of angles and triangles.	

SCO: **G1 – Derive proofs that involve the properties of angles and triangles.** [CN, R, V]

Students who have achieved this outcome should be able to:

A. Generalize, using inductive reasoning, the relationship between pairs of angles formed by transversals and parallel lines, with or without technology.

B. Justify, using deductive reasoning, properties of angles formed by transversals and parallel lines, including the sum of the angles in a triangle.

C. Generalize, using inductive reasoning, a rule for the relationship between the sum of the interior angles and the number of sides, *n*, in a polygon, with or without technology.

D. Identify and correct errors in a given proof of a property involving angles.

E. Verify, with examples, that if lines are not parallel, the angle properties do not apply.

Note: It is intended that deductive reasoning be limited to direct proof.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A E)

2.2 (B D)

2.3 (B D)

2.4 (C D)

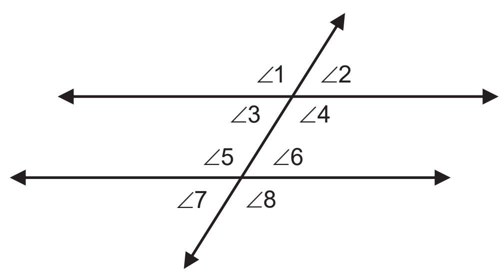
[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: G1 – Derive proofs that involve the properties of angles and triangles. [CN, R, V]

Elaboration

When a transversal line intersects a pair of parallel lines,

- the corresponding angles are equal: $\angle 1 = \angle 5,\ \angle 2 = \angle 6,\ \angle 3 = \angle 7,\ \angle 4 = \angle 8$
- the alternate interior angles are equal: $\angle 3 = \angle 6,\ \angle 4 = \angle 5$
- the alternate exterior angles are equal: $\angle 1 = \angle 8,\ \angle 2 = \angle 7$
- the interior angles on the same side of the transversal are supplementary: $\angle 3 + \angle 5 = 180^0,\ \angle 4 + \angle 6 = 180^0$



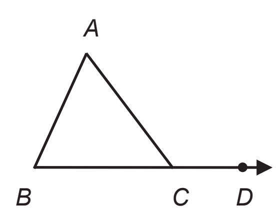
Conversely, when a transversal intersects a pair of lines such that:

- the corresponding angles are equal
- the alternate interior angles are equal
- the alternate exterior angles are equal or
- the interior angles on the same side of the transversal are supplementary

then the lines are parallel. When a transversal intersects a pair of non-parallel lines, none of the above relationships is true.

The following properties of angles in triangles can be proved using other properties that have already been proved:

- In any triangle, the sum of the measures of the interior angles is 180^0: $\angle ABC + \angle BAC + \angle ACB = 180^0$
- The measure of any exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles: $\angle ACD = \angle ABC + \angle BAC$



The following properties of angles in polygons can be proved using other angle properties that have already been proved:

- The sum of the measures of the interior angles of a convex polygon with n sides can be expressed as $180^0(n-2)$.
- The measure of each interior angle of a regular polygon is $\frac{180^0(n-2)}{n}$.
- The sum of the measures of the exterior angles of any convex polygon is 360^0.

MAT521A – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.	**G2** Solve problems that involve the properties of angles and triangles.	

SCO: **G2 – Solve problems that involve the properties of angles and triangles.** **[CN, PS, V]**

Students who have achieved this outcome should be able to:

A. Determine the measures of angles in a diagram that involves parallel lines, angles, and triangles, and justify the reasoning.

B. Identify and correct errors in a given solution to a given problem that involves the measures of angles.

C. Solve a contextual problem that involves angles or triangles.

D. Determine if lines are parallel, given the measure of an angle at each intersection formed by the lines and a transversal.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B C D)

2.2 (A B C D)

2.3 (A B C)

2.4 (B C)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: G2 – Solve problems that involve the properties of angles and triangles. [CN, PS, V]

Elaboration

When solving problems involving the properties of angles and triangles, it is important to begin by noting all of the angle relationships that exist from the diagram of the given context. Then, use those relationships to find the connection to the particular question that is being asked in the problem.

MAT521A – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.	**G3** Solve problems that involve the cosine law and the sine law.	

SCO: **G3 – Solve problems that involve the cosine law and the sine law.** [CN, PS, R]

Students who have achieved this outcome should be able to:

A. Draw a diagram to represent a problem that involves cosine law or sine law.

B. Solve a problem involving the cosine law that requires the manipulation of a formula.

C. Solve a problem involving the sine law that requires the manipulation of a formula.

D. Solve a contextual problem that involves the sine law or the cosine law.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.1 (A)

3.2 (A B C D)

3.3 (A B C D)

3.4 (A C D)

4.2 (A B C D)

4.4 (A B C D)

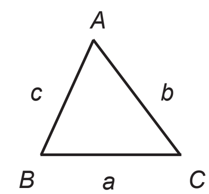
[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: **G3 – Solve problems that involve the cosine law and the sine law.** **[CN, PS, R]**

Elaboration

The ratios of $\dfrac{\text{length of opposite side}}{\sin(\text{angle})}$ are equivalent for all three side-angle pairs in a triangle. As a result, in any triangle, ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ or } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



This relationship is known as the sine law.

The sine law can be used to solve a problem modelled by any triangle when the following information is known:

- two sides and an angle opposite a known side.
- two angles and any side.

If the measures of two angles are known, the third angle can be found by using the property that the three angles in a triangle must add up to 180^0.

The cosine law can be used to determine an unknown side length or angle measure in any triangle. The three versions of the cosine law for triangle *ABC* are:

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

The cosine law can be used to solve a problem modelled by a triangle when the following information is known:

- two sides and the contained angle.
- all three sides.

When solving any problem involving a triangle, it is important to draw a labelled diagram, as the diagram will help determine which strategy to use when solving the problem.

Please note, it is not the intention of this outcome to address the ambiguous case of the sine law.

LOGICAL REASONING

SPECIFIC CURRICULUM OUTCOMES

LR1 – Analyse and prove conjectures using inductive and deductive reasoning.

LR2 – Analyse puzzles and games that involve spatial reasoning, using problem-solving strategies.

MAT521A – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
	LR1 Analyse and prove conjectures, using inductive and deductive reasoning.	**LR3** Solve problems that involve conditional statements.

SCO: **LR1 – Analyse and prove conjectures using inductive and deductive reasoning.**
[C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Make conjectures by observing patterns and identifying properties and justify the reasoning.
B. Explain why inductive reasoning may lead to a false conjecture.
C. Compare, using examples, inductive and deductive reasoning.
D. Provide and explain a counterexample to disprove a given conjecture.
E. Prove algebraic number tricks.
F. Determine if a given argument is valid and justify the reasoning.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (A)
1.2 (B)
1.3 (D)
1.4 (C E F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: LR1 – Analyse and prove conjectures, using inductive and deductive reasoning. [C, CN, PS, R]

Elaboration

Inductive reasoning involves looking at specific examples. By observing patterns and identifying properties in these examples, one may be able to make a general conclusion, which can be stated as a conjecture. However, inductive reasoning alone will not prove a conjecture.

Once a single counterexample has been found to a conjecture, it has been disproved. This means that the conjecture is invalid. In certain cases, a counterexample may also be used to revise a conjecture.

Deductive reasoning involves starting with general assumptions that are known to be true and, through logical reasoning, arriving at a specific conclusion. A conjecture has been proved only when it has been shown to be true for every possible case or example. This is accomplished by creating a proof that involves general cases. When the principles of deductive reasoning are applied properly, one can be sure that the conclusion which is drawn is valid. It is important to note that a demonstration involving an example is not a proof.

A property that is often useful in deductive reasoning is the transitive property. It can be stated as, "Things that are equal to the same thing are equal to each other." In symbols, it can be written as, "If $a = b$ and $b = c$, then $a = c$."

When constructing proofs, it is important to note that a single error in reasoning will break down the logical argument of a deductive proof. This will result in an invalid conclusion, or a conclusion that is not supported by the proof. Two common errors made by students when constructing proofs are:

- division by an expression that is equal to zero.
- assuming a result that follows from what one is trying to prove, also called circular reasoning.

It is also important to realize that the reason one writes a proof is so that others can read and understand it. After a proof has been written, if someone who reads it gets confused or does not understand the logical arguments in the proof, then it needs to be more clearly written.

MAT521A – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
	LR2 Analyse puzzles and games that involve spatial reasoning, using problem-solving strategies.	**LR1** Analyse puzzles and games that involve numerical and logical reasoning, using problem-solving strategies.

SCO: **LR2 – Analyse puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, V]**

Students who have achieved this outcome should be able to:

A. Determine, explain and verify a strategy to solve a puzzle or to win a game; *e.g.*:

- guess and check;
- look for a pattern;
- make a systematic list;
- draw or model;
- eliminate possibilities;
- simplify the original problem;
- work backward;
- develop alternate approaches.

B. Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

Note: It is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction and similar puzzles and games.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.7 (A B)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: **LR2 – Analyse puzzles and games that involve spatial reasoning, using problem-solving strategies.**
[CN, PS, R, V]

Elaboration

Both inductive and deductive reasoning are useful for determining a strategy to solve a puzzle or win a game. Inductive reasoning is useful when analysing games and puzzles that require recognizing patterns or creating a particular order. Deductive reasoning is useful when analysing games and puzzles that require inquiry and discovery to complete.

STATISTICS

SPECIFIC CURRICULUM OUTCOMES

S1 – Demonstrate an understanding of normal distribution, including:
- **standard deviation;**
- ***z*-scores.**

S2 – Interpret statistical data, using:
- **confidence intervals;**
- **confidence levels;**
- **margin of error.**

MAT521A – Topic: Statistics (S)
GCO: Develop statistical reasoning.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
	S1 Demonstrate an understanding of normal distribution, including: • standard deviation; • *z*-scores.	**P1** Interpret and assess the validity of odds and probability statements. **P2** Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events. **P3** Solve problems that involve the probability of two events.

SCO: **S1 – Demonstrate an understanding of normal distribution, including:**

- **standard deviation;**
- ***z*-scores.**

[CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Explain, using examples, the meaning of standard deviation.

B. Calculate, using technology, the population standard deviation of a data set.

C. Explain, using examples, the properties of a normal curve, including the mean, median, mode, standard deviation, symmetry, and area under the curve.

D. Determine if a data set approximates a normal distribution and explain the reasoning.

E. Compare the properties of two or more normally distributed data sets.

F. Explain, using examples that represent multiple perspectives, the application of standard deviation for making decisions in situations such as those involving warranties, insurance, or opinion polls.

G. Solve a contextual problem that involves the interpretation of standard deviation.

H. Determine, with or without technology, and explain the *z*-score for a given value in a normally distributed data set.

I. Solve a contextual problem that involves normal distribution.

J. Explore the similarities and differences between two sets of data.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.1 (J)

5.3 (A B F G)

5.4 (C D E I)

5.5 (H I)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: **S1 – Demonstrate an understanding of normal distribution, including:**
- **standard deviation;**
- ***z*-scores.**

[CN, PS, T, V]

Elaboration

Measures of central tendency (mean, median, mode) are not always sufficient to represent or compare sets of data. Inferences can also be drawn from numerical data by examining how the data is distributed around the mean or the median. When comparing sets of data, it is important that the data be organized in a systematic way, in order to look at similarities and differences between the sets of data.

To determine how scattered or clustered the data in a set is, determine the mean of the data and compare each value to the mean. The standard deviation, σ, is a measure of the dispersion of the data about the mean. The mean and standard deviation can be determined whether or not the data is grouped.

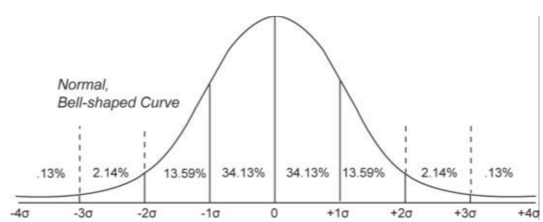
When the data is concentrated close to the mean, the standard deviation is low. When the data is spread far from the mean, the standard deviation is high. As a result, standard deviation is a useful statistic to compare the dispersion between two, or among more, sets of data.

When determining the standard deviation, σ, for a set of data, the following process is followed.

- The square of the deviation of each data value (or the midpoint of the interval) from the mean is determined: $\left(x-\bar{x}\right)^2$.
- The mean of the squares of the deviations of all the data values is determined.
- The square root of the mean of the squares of the deviations is determined. This is the standard deviation.

Graphing a set of grouped data can help one determine whether the shape of the frequency polygon can be approximated by a normal distribution. The properties of a normal distribution are:

- The graph is symmetrical. The mean, median, and mode are equal and fall at the line of symmetry.
- The normal curve is shaped like a bell, peaking in the middle, sloping down toward the sides, and approaching zero at the extremes.
- About 68% of the data is within one standard deviation of the mean, about 95% of the data is within two standard deviations of the mean, and about 99.7% of the data is within three standard deviations of the mean.



- The area under the curve can be considered as one unit since it represents 100% of the data.

The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1. The area under the curve of a standard normal distribution is 1. Data can be compared from different normally distributed sets by using *z*-scores. This process will convert any normal distribution to a standard normal distribution.

A *z*-score indicates the number of standard deviations that a data value lies from the mean. It is calculated using the formula

$$z=\frac{x-\mu}{\sigma}$$

A positive *z*-score indicates that the data value lies above the mean. A negative *z*-score indicates that the data value lies below the mean.

MAT521A – Topic: Statistics (S)
GCO: Develop statistical reasoning.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
	S2 Interpret statistical data, using: • confidence intervals; • confidence levels; • margin of error.	

SCO: **S2 – Interpret statistical data, using:**

- **confidence intervals;**
- **confidence levels;**
- **margin of error.**

[C, CN, R]

Students who have achieved this outcome should be able to:

A. Explain, using examples, how confidence levels, margins of error and confidence intervals may vary depending on the size of the random sample.

B. Explain, using examples, the significance of a confidence interval, margin of error or confidence level.

C. Make inferences about a population from sample data, using given confidence intervals, and explain the reasoning.

D. Provide examples from print or electronic media in which confidence intervals and confidence levels are used to support a particular position.

E. Interpret and explain confidence intervals and margins of error, using examples found in print or electronic media.

F. Support a position by analysing statistical data presented in the media.

Note: It is intended that the focus of this outcome be on interpretation of data rather than on statistical calculations.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.6 (A B C D E F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: S2 – Interpret statistical data, using:
- **confidence intervals;**
- **confidence levels;**
- **margin of error.**

[C, CN, R]

Elaboration

It is often impractical, if not impossible, to obtain data for a complete population. Instead, a random sample of a population is taken, and the mean and standard deviation of the data in the sample are determined. This information is then used to make predictions about the population. When data approximates a normal distribution, a confidence interval indicates the range in which the mean of a sample of data would be expected to lie if other samples of the same size were taken, to a stated degree of accuracy. This confidence interval can be then used to estimate the range of the mean for the population. Sample size, confidence level, and population size determine the size of the confidence interval for a given confidence level.

A confidence interval is expressed as a survey or poll result, plus or minus the margin of error. The margin of error increases as the confidence level increases (with a constant sample size). The sample size that is needed also increases as the confidence level increases (with a constant margin of error). The sample size affects the margin of error. A larger sample results in a smaller margin of error, assuming that the same confidence level is required.

RELATIONS AND FUNCTIONS

SPECIFIC CURRICULUM OUTCOMES

RF1 – Model and solve problems that involve systems of linear inequalities in two variables.

RF2 – Demonstrate an understanding of the characteristics of quadratic functions, including:
- **vertex;**
- **Intercepts;**
- **domain and range;**
- **axis of symmetry.**

MAT521A – Topic: Relations and Functions (RF)
GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.	**RF1** Model and solve problems that involve systems of linear inequalities in two variables.	

SCO: **RF1 – Model and solve problems that involve systems of linear inequalities in two variables. [CN, PS, T, V]**

Students who have achieved this outcome should be able to:

A. Model a problem, using a system of linear inequalities in two variables.

B. Graph the boundary line between two half planes for each inequality in a system of linear inequalities and justify the choice of solid or broken lines.

C. Determine and explain the solution region that satisfies a linear inequality, using a test point when given a boundary line.

D. Determine, graphically, the solution region for a system of linear inequalities, and verify the solution.

E. Explain, using examples, the significance of the shaded region in the graphical solution of a system of linear inequalities.

F. Solve an optimization problem, using linear programming.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (C)

6.2 (E)

6.3 (A B D E)

6.4 (F)

6.5 (F)

6.6 (F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: **RF1 – Model and solve problems that involve systems of linear inequalities in two variables.**
[CN, PS, T, V]

Elaboration

When a linear inequality in two variables is represented graphically, its boundary divides the Cartesian plane into two half-planes. One of these half-planes represents the solution set of the linear inequality, which may or may not include points on the boundary itself.

To graph a linear inequality in two variables, these steps may be followed:

- Graph the boundary of the solution region.
  - If the linear inequality includes the possibility of equality ($\le$ or $\ge$), and the solution set is continuous, draw a solid line to show that all points on the boundary are included.
  - If the linear inequality includes the possibility of equality ($\le$ or $\ge$), and the solution set is discrete, draw the points on the boundary that satisfy the corresponding equation.
  - If the linear inequality excludes the possibility of equality (< or >), draw a dashed line to show that the points on the boundary are not included.
- Choose a test point that is on one side of the boundary.
  - Substitute the coordinates of the test point into the linear inequality.
  - If the test point is a solution to the linear inequality, shade the half-plane that contains the point. Otherwise, shade the other half-plane.
  - Use $(0,0)$ as the test point to simplify calculations if it does not lie on the boundary.

An alternate method used to determine which half-plane to shade is to solve the original linear inequality for *y*, keeping the *y* on the left-hand side of the inequality. If the resulting inequality includes a greater than sign $(> \text{ or } \ge)$, then the top half is shaded. If the resulting inequality includes a less than sign $(< \text{ or } \le)$, then the bottom half is shaded. Please note that if the boundary line is vertical, a less than sign indicates to shade the left side of the boundary line, and a greater than sign indicates to shade the right side of the boundary line.

When sketching the graph of a system of linear inequalities, the intersection point may or may not be included, depending on the types of linear inequalities in the system. Use an open dot to show that an intersection point of a system's boundaries is excluded from the solution set. An intersection point is excluded if either boundary is a dashed line. Use a closed dot to show that an intersection point of a system's boundaries is included in the solution set. An intersection point is included only if both boundaries are solid lines.

The solution to an optimization problem is usually found at one of the vertices of the feasible region. To determine the optimal solution to an optimization problem using linear programming, follow these steps:

- Identify the quantity that must be optimized. Look for key words, such as *maximize* or *minimize*, *largest* or *smallest*, or *greatest* or *least*.
- Define the variables that affect the quantity to be optimized. Identify any restrictions on these variables.
- Write a system of linear inequalities to describe all the constraints of the problem. Graph the system.
- Write an objective function to represent the relationship between the variables and the quantity to be optimized.
- Evaluate the objective function at each vertex of the feasible region of the system of linear inequalities.
- Compare the results and choose the desired solution.
- Verify that the solution(s) satisfies the constraints of the problem situation.

MAT521A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521A	GRADE 12 – MAT621A
RF5 Determine the characteristics of the graphs of linear relations, including the: • intercepts; • slope; • domain; • range.	**RF2** Demonstrate an understanding of the characteristics of quadratic functions, including: • vertex; • Intercepts; • domain and range; • axis of symmetry.	**RF1** Represent data, using polynomial functions (of degree $\leq$ 3), to solve problems. **RF2** Represent data, using exponential and logarithmic functions, to solve problems. **RF3** Represent data, using sinusoidal functions, to solve problems.

SCO: **RF2 – Demonstrate an understanding of the characteristics of quadratic functions, including:**

- **vertex;**
- **intercepts;**
- **domain and range;**
- **axis of symmetry.**

[CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Determine, with or without technology, the intercepts of the graph of a quadratic function.

B. Determine, using the quadratic formula, the roots of a quadratic equation.

C. Explain the relationships among the roots of an equation, the zeros of the corresponding function, and the *x*-intercepts of the graph of the function.

D. Explain, using examples, why the graph of a quadratic function may have zero, one or two *x*-intercepts.

E. Determine, with or without technology, the coordinates of a vertex of the graph of a quadratic function.

F. Determine the equation of the axis of symmetry of the graph of a quadratic function, given the *x*-intercepts of the graph.

G. Determine the coordinates of the vertex of the graph of a quadratic function, given the equation of the function and the axis of symmetry, and determine if the *y*-coordinate of the vertex is a maximum or a minimum.

H. Determine the domain and range of a quadratic function.

I. Sketch the graph of a quadratic function.

J. Solve a contextual problem that involves the characteristics of a quadratic function.

Note: It is intended that completion of the square not be required.

Section(s) in Foundations of Mathematics 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.1 (I)
7.2 (A E F G H I J)
7.3 (A C D I J)
7.6 (A E F G H J)
7.7 (B J)
7.8 (L)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: **RF2 – Demonstrate an understanding of the characteristics of quadratic functions, including:**

- **vertex;**
- **intercepts;**
- **domain and range;**
- **axis of symmetry.**

[CN, PS, T, V]

Elaboration

The standard form of a quadratic function is $y = ax^2 + bx + c$ where $a \neq 0$. The degree of all quadratic functions is two. The graph of any quadratic function is a parabola with a single vertical line of symmetry, through its vertex. The domain of all quadratic functions is the set of all real numbers and the range depends on whether the graph opens upward or downward. If a > 0, the graph of the parabola opens upward, its vertex will be at a minimum, and the range is $\{y | y \geq k, y \in R\}$. If a < 0, the graph of the parabola opens downward, its vertex will be at a maximum, and the range is $\{y | y \leq k, y \in R\}$.

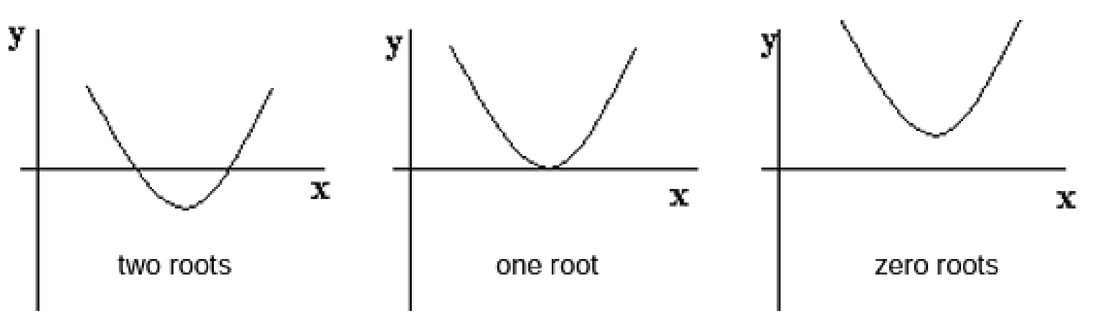
A quadratic function may also be written in vertex form. The following table highlights the characteristics of quadratic functions written in both standard and vertex form. As one can see, certain characteristics are more easily found by writing the quadratic function in certain forms. Those that are more difficult to find directly from the equation are left blank in the table. However, all these characteristics can be found with the help of some algebra.

CHARACTERISTICS OF QUADRATIC FUNCTIONS		
CHARACTERISTIC	**STANDARD FORM**	**VERTEX FORM**
Form	$f(x) = ax^2 + bx + c$	$f(x) = a(x-h)^2 + k$
Vertex		(h, k)
Axis of Symmetry		x = h
y-intercept	(0, c)	

The x-intercepts for a quadratic function in standard form can be determined by using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Since $-\frac{b}{2a}$ is the first term in the quadratic formula, the formula $h = -\frac{b}{2a}$ may be used to determine the axis of symmetry and the h value of the vertex for a quadratic in standard form. The *k* value of the vertex can be found by substituting the h value into the original function and simplifying the result.

The y-intercept of a quadratic function in vertex form can be determined by substituting 0 for x into the function and simplifying the result.

A quadratic equation can be solved by graphing the corresponding quadratic function. The roots of a quadratic equation are the x-intercepts of the graph of the corresponding quadratic function. A quadratic equation may have zero, one, or two roots, because the graph of the corresponding parabola may intersect the x-axis in zero, one, or two places, as is illustrated below:



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