Prince Edward Island
CANADA
Education and Early Years

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

MAT521B

Acknowledgments

The Department of Education and Early Years of Prince Edward Island gratefully acknowledges the contributions of the following groups and individuals toward the development of the *Prince Edward Island MAT521B Mathematics Curriculum Guide*:

- J. Blaine Bernard,
  Secondary Mathematics Specialist,
  Department of Education and
  Early Childhood Development

- The 2011-2012 MAT521B pilot teachers:

  Stephanie Campbell,
  Westisle Composite High School

  Amanda Cooke,
  Westisle Composite High School

  Alden Deroche,
  Colonel Gray Senior High School

  Karen Malone,
  Montague Regional High School

  Alan McAlduff,
  Westisle Composite High School

  Kelly O'Connell,
  Colonel Gray Senior High School

- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education

- Alberta Education

Prince Edward Island
Department of Education and Early Years
Holman Centre
250 Water Street, Suite 101
Summerside, Prince Edward Island
Canada C1N 1B6
Implemented 2012
Revised 2024

Table of Contents

Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for Grades 10-12 Mathematics* (2008) has been adopted as the basis for a revised high school mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential Graduation Learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

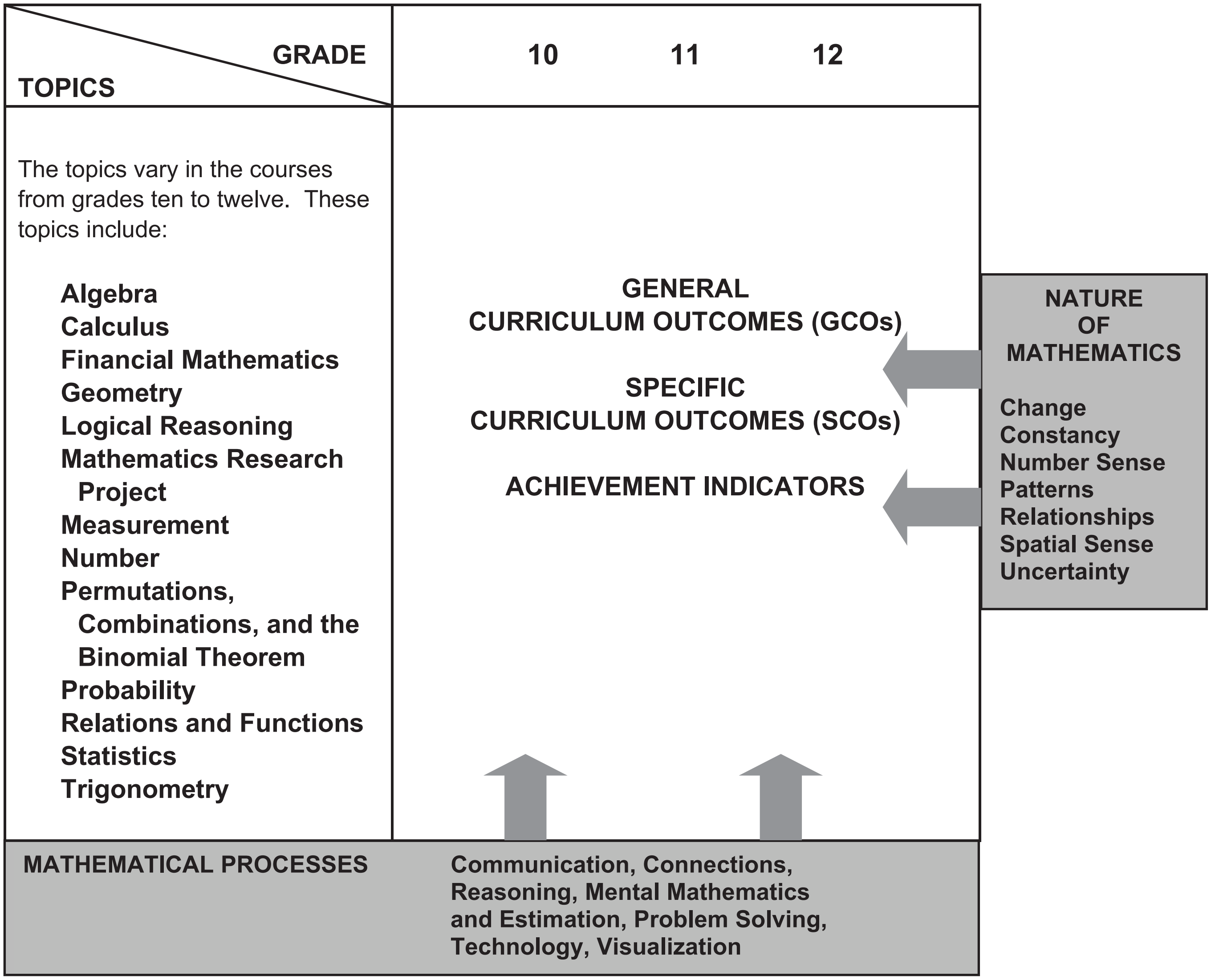
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

➢ Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for 10-12 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



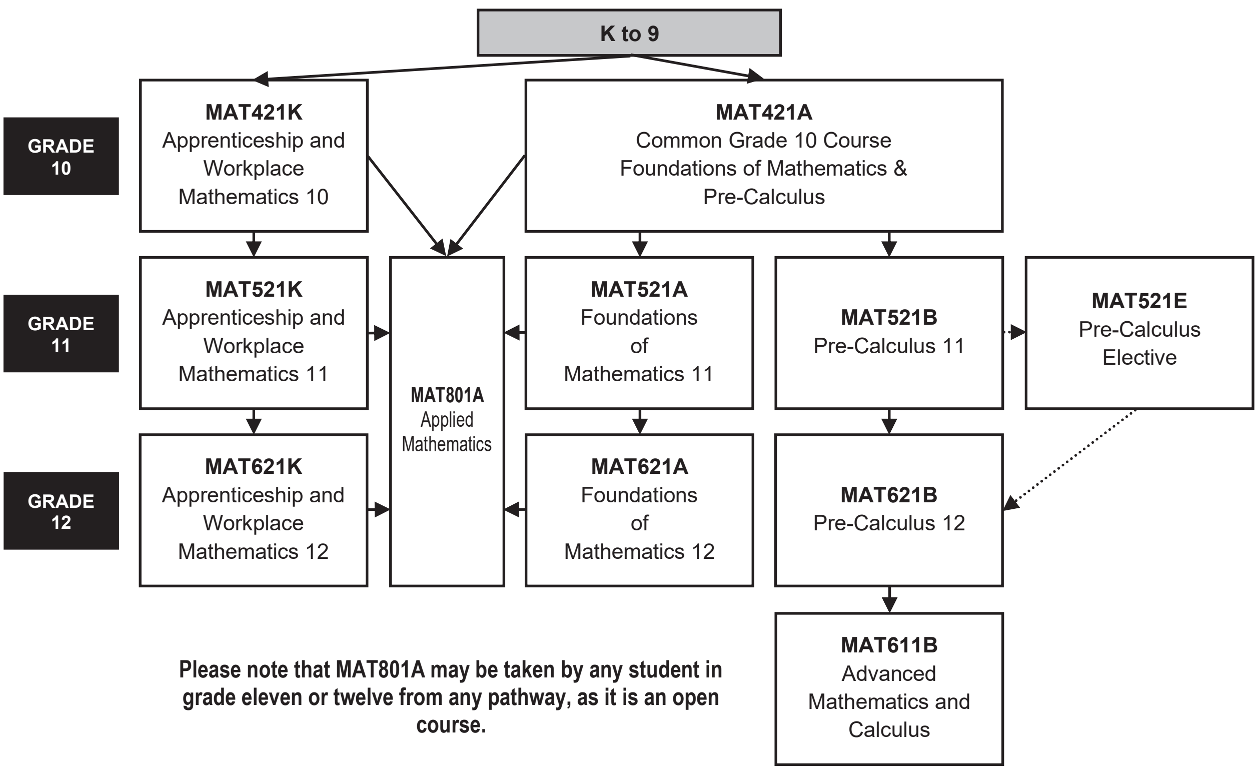
The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into a number of topics, as described above. These topics are not intended to be discrete units of instruction. The integration of outcomes across topics makes mathematical experiences meaningful. Students should make the connections among concepts both within and across topics. Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.

- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Pathways and Topics

The Prince Edward Island 10-12 mathematics curriculum includes pathways with corresponding topics rather than strands, which are found in the Prince Edward Island K-9 mathematics curriculum. Three pathways are available: **Apprenticeship and Workplace Mathematics**, **Foundations of Mathematics**, and **Pre-Calculus**. A common grade ten course (Foundations of Mathematics and Pre-Calculus, Grade 10) is the starting point for the Foundations of Mathematics pathway and the Pre-Calculus pathway. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings. These pathways are illustrated in the diagram below:



The goals of all three pathways are to provide the prerequisite knowledge, skills, understandings, and attitudes for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study or for direct entry into the work force. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Apprenticeship and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics, and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, algebra and number, logical reasoning, relations and functions, statistics, probability, and a mathematics research project.

Pre-Calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, combinatorics, and introductory calculus.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

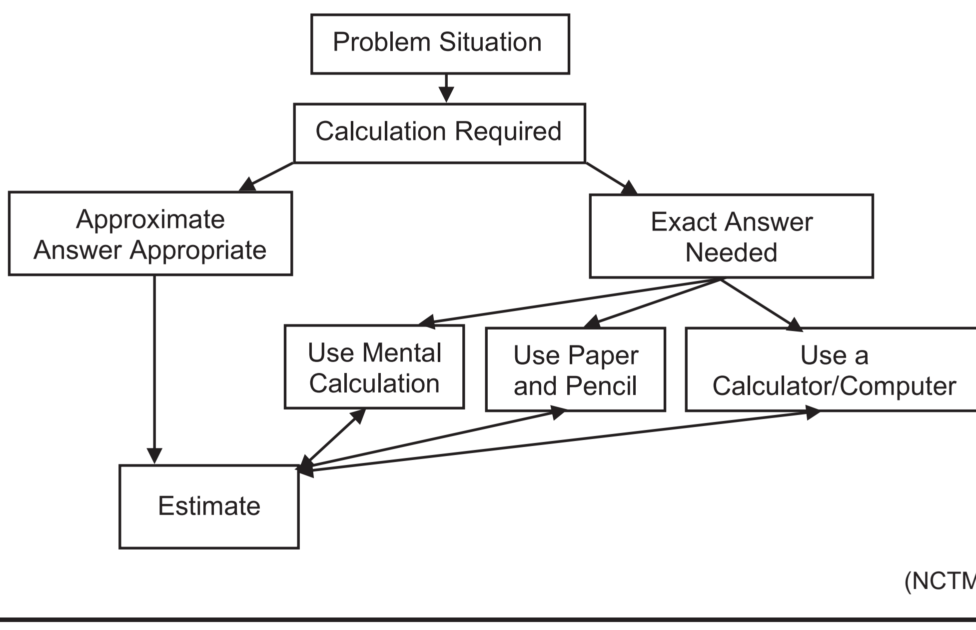
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?" or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;

- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.

- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all topics in mathematics and it is important that connections are made among topics. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical topics. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education" (p.60). The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate “communicating to learn mathematics and learning to communicate mathematically” (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

➢ Inquiry-Based Learning and Project Based Learning

Inquiry-based learning (IBL) allows students to explore, investigate, and construct new meaning from prior knowledge and from new information that is retrieved from other sources. It is not linear in nature, but promotes a continual looping back and forth throughout the process as students gather and process new information, redirect their inquiries, and continue through the process. Mathematical inquiry will require students to practise and refine their critical and creative-thinking skills. The terms *inquiry* and *research* are often used interchangeably within an educational context. While research often becomes the end-result of an inquiry process, it is the process itself that should be emphasized within an educational context. More information regarding the development of a mathematics research project is included in the appendix at the end of this document.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, teaching has been effective, or how best to address student learning needs. The quality of assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

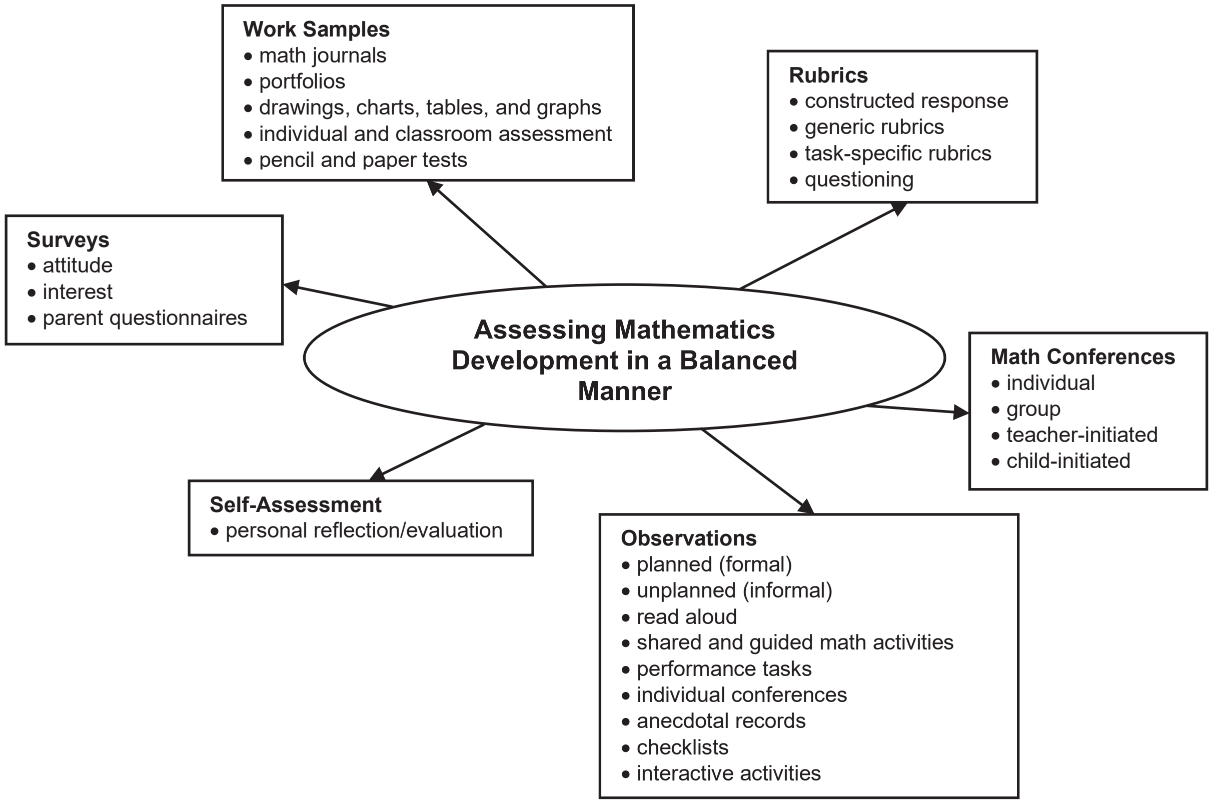
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;

- to provide the basis for sound decision-making about next steps in a student’s learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children’s progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, phone calls, and electronic methods.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student’s performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;

- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he or she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island high school mathematics curriculum are organized into a number of topics across the grades from ten to twelve. Each topic has associated with it a general curriculum outcome (GCO). They are overarching statements about what students are expected to learn in each topic from grades ten to twelve.

Topic	General Curriculum Outcome (GCO)
Algebra (A)	Develop algebraic reasoning.
Algebra and Number (AN)	Develop algebraic reasoning and number sense.
Calculus (C)	Develop introductory calculus reasoning.
Financial Mathematics (FM)	Develop number sense in financial applications.
Geometry (G)	Develop spatial sense.
Logical Reasoning (LR)	Develop logical reasoning.
Mathematics Research Project (MRP)	Develop an appreciation of the role of mathematics in society.
Measurement (M)	Develop spatial sense and proportional reasoning. (***Foundations of Mathematics and Pre-Calculus***)
	Develop spatial sense through direct and indirect measurement. (***Apprenticeship and Workplace Mathematics***)
Number (N)	Develop number sense and critical thinking skills.
Permutations, Combinations and Binomial Theorem (PC)	Develop algebraic and numeric reasoning that involves combinatorics.
Probability (P)	Develop critical thinking skills related to uncertainty.
Relations and Functions (RF)	Develop algebraic and graphical reasoning through the study of relations.
Statistics (S)	Develop statistical reasoning.
Trigonometry (T)	Develop trigonometric reasoning.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific curriculum outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding topic and general curriculum outcome;
- the scope and sequence of the specific curriculum outcome(s) from grades ten to twelve which correspond to this SCO;
- the specific curriculum outcome, with a list of achievement indicators;

- a list of the sections in *Pre-Calculus 11* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

ALGEBRA AND NUMBER

SPECIFIC CURRICULUM OUTCOMES

AN1 – Demonstrate an understanding of the absolute value of real numbers.

AN2 – Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands.

AN3 – Solve radical equations (limited to square roots).

AN4 – Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).

AN5 – Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).

AN6 – Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials).

MAT521B – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
	AN1 Demonstrate an understanding of the absolute value of real numbers.	

SCO: **AN1 – Demonstrate an understanding of the absolute value of real numbers.** [R, V]

Students who have achieved this outcome should be able to:

A. Determine the distance of two real numbers of the form $\pm a$, $a \in R$, from 0 on a number line, and relate this to the absolute value of a $(|a|)$.

B. Determine the absolute value of a real number.

C. Determine the absolute value of a numerical expression.

D. Compare and order the absolute values of real numbers in a given set.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.1 (A B C D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: AN1 – Demonstrate an understanding of the absolute value of real numbers. [R, V]

Elaboration

The absolute value of a real number, *a*, indicates the distance that *a* is from the origin, zero, on a number line, regardless of direction. It is denoted by $|a|$ and is defined as follows:

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

The absolute value of a real number and its opposite are equal, as these numbers lie at the same distance from zero on opposite sides of the number line. Since the absolute value of a number indicates a distance, it is never negative, that is $|a| \geq 0$. Strictly speaking, it is incorrect to state that the absolute value of a real number is always positive, since the absolute value of zero is zero, which is not a positive number.

To determine the absolute value of a real number, it is simply necessary to write the number without its sign. To determine the absolute value of a numerical expression, it is necessary to first simplify the expression within the absolute value bars. After simplifying, find the absolute value of the resulting number.

If an expression contains an absolute value expression within it, first simplify the expression within the absolute value bars, find its absolute value, and then simplify the remaining expression. For example,

$$\begin{aligned} 10 - 5|1 - 3(2)| &= 10 - 5|1 - 6| \\ &= 10 - 5|-5| \\ &= 10 - 5(5) \\ &= 10 - 25 \\ &= -15 \end{aligned}$$

Before ordering the absolute values of real numbers in a given set, it is important to first find the absolute value of each of the numbers in the set. Essentially, this means that we are ordering the magnitudes of the numbers and ignoring the signs on the numbers.

MAT521B – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
AN1 Demonstrate an understanding of factors of whole numbers by determining the: • prime factors; • greatest common factor; • least common multiple; • square root; • cube root.	**AN2** Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands.	

SCO: **AN2 – Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands.** **[CN, ME, PS, R, T]**

Students who have achieved this outcome should be able to:

A. Express an entire radical with a numerical radicand as a mixed radical.

B. Express a mixed radical with a numerical radicand as an entire radical.

C. Perform one or more operations to simplify radical expressions with numerical or variable radicands.

D. Rationalize the denominator of a rational expression with monomial or binomial denominators.

E. Explain, using examples, that $(-x)^2 = x^2$, $\sqrt{x^2} = |x|$ and $\sqrt{x^2} \neq \pm x$; *e.g.*: $\sqrt{9} \neq \pm 3$.

F. Identify the values of the variable for which a given radical expression is defined.

G. Solve a problem that involves radical expressions.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.1 (A B E F G)

5.2 (C D E G)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: AN2 – Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. **[CN, ME, PS, R, T]**

Elaboration

When simplifying a radical expression with an even index, the solution cannot be negative. As a result, $\sqrt{x^2} = |x|$, and $\sqrt{9} = 3$, as the index is 2 in each of these expressions. It is therefore incorrect to say that $\sqrt{9} = \pm 3$.

A radical expression is in simplest form if:

- the radicand does not have a perfect root factor;
- the radicand does not contain a fraction;
- the denominator does not contain a radical
  - If the denominator is a monomial radical expression, multiply the numerator and denominator by a radical that will create a perfect root on the denominator.
  - If the denominator is a binomial radical expression containing square roots, multiply numerator and denominator by the conjugate of the denominator.

Students should understand that operations involving radical expressions are similar to operations involving polynomials. A comparison between the operations of radicals and polynomials is shown in the following table:

RADICALS	POLYNOMIALS	COMMENTS
$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$	$2x + 5x = 7x$	When adding radical expressions, the radicands must be the same. To add, simply add the coefficients.
$7\sqrt{2} - 4\sqrt{2} = 3\sqrt{2}$	$7y - 4y = 3y$	When subtracting radical expressions, the radicands must be the same. To subtract, simply subtract the coefficients.
$\left(3\sqrt{5}\right)\left(2\sqrt{3}\right) = 6\sqrt{15}$	$(3x)(2y) = 6xy$	When multiplying radical expressions, multiply the coefficients and the radicands, respectively.
$\frac{24\sqrt{30}}{8\sqrt{5}} = 3\sqrt{6}$	$\frac{24xy}{8x} = 3y$	When dividing radical expressions, divide the coefficients and the radicands, respectively.

When a radicand contains variables, consider the index and the radicand when determining the values of the variable that make the expression a real number:

- If the index is an even number, the radicand must be non-negative, that is, greater than or equal to zero.
- If the index is an odd number, the radicand may be any real number.

MAT521B – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.	**AN3** Solve radical equations (limited to square roots).	**RF13** Graph and analyse radical functions (limited to functions involving one radical).

SCO: **AN3 – Solve radical equations (limited to square roots). [C, PS, R]**

Students who have achieved this outcome should be able to:

A. Determine any restrictions on the values for the variable in a radical equation.

B. Determine the roots of a radical equation algebraically, and explain the process used to solve the equation.

C. Verify, by substitution, that the values determined in solving a radical equation algebraically are not extraneous roots of the equation.

Note: It is intended that the equations will have no more than two radicals.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.3 (A B C)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: AN3 – Solve radical equations (limited to square roots). [C, PS, R]

Elaboration

When solving radical equations, begin by isolating one of the radical expressions. To eliminate a square root, square both sides of the equation and solve the remaining equation. After solving, if the equation still contains a radical expression, isolate it and repeat the process.

It is possible that squaring may introduce extraneous roots, so it is necessary that students always check for extraneous roots after they have found possible solutions to a radical equation, as there is no easy way to predict whether possible solutions will work in the original equation. For example,

$$\sqrt{x-4}=x-10$$
$$\left(\sqrt{x-4}\right)^2=(x-10)^2$$
$$x-4=x^2-20x+100$$
$$0=x^2-21x+104$$
$$0=(x-8)(x-13)$$
$$x-8=0 \text{ or } x-13=0$$
$$x=8 \quad \Big| \quad x=13$$

Check:

$$\begin{array}{c|c} \multicolumn{2}{c}{\sqrt{x-4}=x-10} \\ \hline \sqrt{8-4} & 8-10 \\ \sqrt{4} & -2 \\ 2 & \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{\sqrt{x-4}=x-10} \\ \hline \sqrt{13-4} & 13-10 \\ \sqrt{9} & 3 \\ 3 & \end{array}$$

After checking the possible solutions, we can see that $x=13$ is a solution, but $x=8$ is not.

When determining restrictions on the variables, it is important to remember that denominators cannot be zero and that radicands must be greater than or equal to zero.

MAT521B – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
AN4 Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.	**AN4** Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).	

SCO: **AN4 – Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [C, ME, R]**

Students who have achieved this outcome should be able to:

A. Compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers.

B. Explain why a given value is non-permissible for a given rational expression.

C. Determine the non-permissible values for a rational expression.

D. Determine a rational expression that is equivalent to a given rational expression by multiplying the numerator and denominator by the same factor (limited to a monomial or a binomial), and state the non-permissible values of the equivalent rational expression.

E. Simplify a rational expression.

F. Identify and correct errors in a simplification of a rational expression and explain the reasoning.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (A B C D E F)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: AN4 – Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [C, ME, R]

Elaboration

A rational expression is an algebraic fraction of the form $\frac{p}{q}$, where *p* and *q* are polynomials and $q \neq 0$. The methods that are used to simplify rational numbers may be extended to working with rational expressions. As when working with rational numbers, we simplify by factoring the numerator and denominator and by dividing both expressions by all common factors.

There does, however, exist one important difference between working with rational numbers and rational expressions. Since rational expressions contain variables, there may be some values of the variables which are non-permissible. These will occur for any value of a variable which makes the denominator equal to zero, as division by zero is undefined.

For example, in the rational expression $\frac{3x-9}{x^2-4}$, $x=-2$ and $x=2$ will each make the denominator equal to zero. As a result, the expression $\frac{3x-9}{x^2-4}$ is undefined at $x=\pm 2$. The numerator, however, of a rational expression may equal zero. For this example, if $x=3$, then the rational expression $\frac{3x-9}{x^2-4}$ has a value of zero.

MAT521B – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
AN4 Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.	**AN5** Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).	

SCO: **AN5 – Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [CN, ME, R]**

Students who have achieved this outcome should be able to:

A. Compare the strategies for performing a given operation on rational expressions to the strategies for performing the same operation on rational numbers.

B. Determine the non-permissible values when performing operations on rational expressions.

C. Determine, in simplified form, the sum or difference of rational expressions with the same denominator.

D. Determine, in simplified form, the sum or difference of rational expressions in which the denominators are not the same and which may or may not contain common factors.

E. Determine, in simplified form, the product or quotient of rational expressions.

F. Simplify an expression that involves two or more operations on rational expressions.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.2 (A B E F)

6.3 (A B C D G)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: AN5 – Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [CN, ME, R]

Elaboration

Performing operations on rational expressions is very similar to performing operations on rational numbers:

- When adding or subtracting rational expressions, a common denominator is required. Then the appropriate operations are performed to the numerators, while the denominator remains the same in the solution.
- When multiplying rational expressions, all numerators and denominators are factored, common factors in numerators and denominators are cancelled, then the remaining factors in the numerators and denominators, respectively, are multiplied to create a simplified expression.
- When dividing rational expressions, we convert division to multiplication by multiplying the first rational expression by the reciprocal of the divisor, which is the rational expression that follows the division sign. Then, we multiply the expressions together as described above.

MAT521B – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.	**AN6** Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials).	**RF14** Graph and analyse rational functions (limited to numerators and denominators that are monomials, binomials or trinomials).

SCO: **AN6 – Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials). [C, PS, R]**

Students who have achieved this outcome should be able to:

A. Determine the non-permissible values for the variable in a rational equation.

B. Determine the solution to a rational equation algebraically, and explain the process used to solve the equation.

C. Explain why a value obtained in solving a rational equation may not be a solution of the equation.

D. Solve problems by modelling a situation using a rational equation.

Note: It is intended that the rational equations be those that can be simplified to linear or quadratic equations.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.4 (A B C D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: AN6 – Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials). [C, PS, R]

Elaboration

Rational equations can be solved by multiplying both sides of the equation by a common denominator. This will eliminate all fractions from the equation. Then, the resulting equation is solved.

However, it is possible that multiplying by a common denominator may introduce extraneous roots. It is, therefore, necessary that students always check for extraneous roots after they have found possible solutions to a rational equation, as there is no easy way to predict whether possible solutions will work in the original equation. For example,

$$\frac{1}{x-2}=\frac{3}{x+2}-\frac{6x}{x^2-4}$$

$$\frac{1}{x-2}=\frac{3}{x+2}-\frac{6x}{(x-2)(x+2)}$$

$$(x-2)(x+2)\left(\frac{1}{x-2}\right)=(x-2)(x+2)\left[\frac{3}{x+2}-\frac{6x}{(x-2)(x+2)}\right]$$

$$1(x+2)=3(x-2)-6x$$

$$x+2=3x-6-6x$$

$$x-3x+6x=-6-2$$

$$4x=-8$$

$$x=-2$$

However, we can see by substituting $x=-2$ into $\frac{6x}{x^2-4}$ that it is a non-permissible solution to the original equation. Therefore, this equation has no solution.

TRIGONOMETRY

SPECIFIC CURRICULUM OUTCOMES

T1 – Demonstrate an understanding of angles in standard position (0^0 to 360^0).

T2 – Solve problems, using the three primary trigonometric ratios for angles from 0^0 to 360^0 in standard position.

T3 – Solve problems, using the cosine law and sine law, including the ambiguous case.

MAT521B – Topic: Trigonometry (T)

GCO: Develop trigonometric reasoning.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.	**T1** Demonstrate an understanding of angles in standard position (0^0 to 360^0).	**T1** Demonstrate an understanding of angles in standard position, expressed in degrees and radians. **T2** Develop and apply the equation of the unit circle.

SCO: **T1 – Demonstrate an understanding of angles in standard position (0^0 to 360^0).** [R, V]

Students who have achieved this outcome should be able to:

A. Sketch an angle in standard position, given the measure of the angle.

B. Determine the reference angle for an angle in standard position.

C. Explain, using examples, how to determine the angles from 0^0 to 360^0 that have the same reference angle as a given angle.

D. Determine the quadrant in which a given angle in standard position terminates.

E. Draw an angle in standard position given any point $P(x, y)$ on the terminal arm of the angle.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B C D E)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

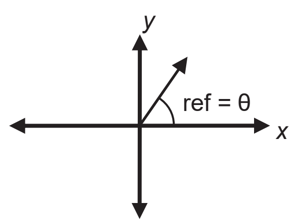
SCO: T1 – Demonstrate an understanding of angles in standard position (0^0 to 360^0). [R, V]

Elaboration

An angle, θ, is in standard position if its initial arm lies on the positive *x*-axis and its vertex is at the origin. If the angle of rotation is counterclockwise, it is positive; if the angle of rotation is clockwise, it is negative.

The reference angle to θ is a positive angle that is greater than 0^0 and less than 90^0. Its initial arm lies on one side of the *x*-axis and its terminal arm lies on the terminal arm of θ. The following diagrams illustrate how to find the reference angle for an angle whose terminal arm lies in each of the four quadrants.

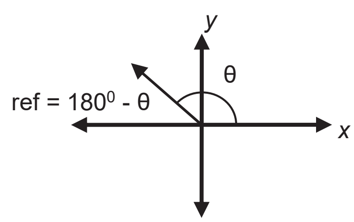
Quadrant I:



If θ is in Quadrant I, then $0^0 < \theta < 90^0$. Since θ has the necessary requirements for a reference angle,

reference angle = θ

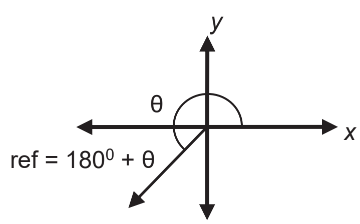
Quadrant II:



If θ is in Quadrant II, then $90^0 < \theta < 180^0$. Since the terminal arm of θ is closer to the negative *x*-axis than the positive *x*-axis and lies above it,

reference angle = $180^0 - \theta$

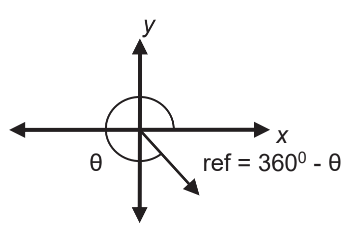
Quadrant III:



If θ is in Quadrant III, then $180^0 < \theta < 270^0$. Since the terminal arm of θ is closer to the negative *x*-axis than the positive *x*-axis and lies below it,

reference angle = $180^0 + \theta$

Quadrant IV:



If θ is in Quadrant IV, then $270^0 < \theta < 360^0$. Since the terminal arm of θ is closer to the positive *x*-axis than the negative *x*-axis and lies below it,

reference angle = $360^0 - \theta$

MAT521B – Topic: Trigonometry (T)

GCO: Develop trigonometric reasoning.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.	**T2** Solve problems, using the three primary trigonometric ratios for angles from 0^0 to 360^0 in standard position.	**T3** Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees. **T4** Graph and analyse the trigonometric functions sine, cosine and tangent to solve problems. **T5** Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians.

SCO: **T2 – Solve problems, using the three primary trigonometric ratios for angles from 0^0 to 360^0 in standard position.** **[C, ME, PS, R, T, V]**

Students who have achieved this outcome should be able to:

A. Determine, using the Pythagorean Theorem or the distance formula, the distance from the origin to a point $P(x,y)$ on the terminal arm of an angle.

B. Determine the value of $\sin\theta$, $\cos\theta$ or $\tan\theta$, given any point $P(x,y)$ on the terminal arm of angle θ.

C. Determine, without the use of technology, the value of $\sin\theta$, $\cos\theta$ or $\tan\theta$, given any point $P(x,y)$ on the terminal arm of angle θ, where $\theta = 0^0$, 90^0, 180^0, 270^0 or 360^0.

D. Determine the sign of a given trigonometric ratio for a given angle, without the use of technology, and explain.

E. Solve, for all values of θ, an equation of the form $\sin\theta = a$ or $\cos\theta = a$, where $-1 \le a \le 1$, and an equation of the form $\tan\theta = a$, where a is a real number.

F. Determine the exact value of the sine, cosine or tangent of a given angle with a reference angle of 30^0, 45^0 or 60^0.

G. Sketch a diagram to represent a problem.

H. Solve a contextual problem, using trigonometric ratios.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A F G)

2.2 (A B C D E F G H)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: T2 – Solve problems, using the three primary trigonometric ratios for angles from 0^0 to 360^0 in standard position. [C, ME, PS, R, T, V]

Elaboration

The primary trigonometric ratios for an angle θ in standard position that has a point $P(x, y)$ on its terminal arm are

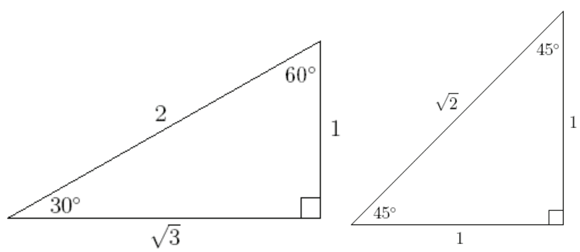
$$\cos\ \theta = \frac{x}{r} \qquad \sin\ \theta = \frac{y}{r} \qquad \tan\ \theta = \frac{y}{x}$$

where $r = \sqrt{x^2 + y^2}$.

The following table shows the signs of the primary trigonometric ratios for an angle θ, in standard position with the terminal arm in the given quadrant.

	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
sin θ	+	+	–	–
cos θ	+	–	–	+
tan θ	+	–	+	–

Exact trigonometric ratios for angles of 30^0, 45^0, and 60^0 can be found by using these special triangles.



The exact values are given in the following table.

	sin θ	cos θ	tan θ
30^0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45^0	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60^0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

If the terminal arm of an angle, θ, in standard position lies on one of the axes of the Cartesian plane, θ is called a quadrantal angle. If $0^0 \le \theta \le 360^0$, the quadrantal angles are 0^0, 90^0, 180^0, 270^0, and 360^0.

MAT521B – Topic: Trigonometry (T)

GCO: Develop trigonometric reasoning.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.	**T3** Solve problems, using the cosine law and sine law, including the ambiguous case.	

SCO: **T3 – Solve problems, using the cosine law and sine law, including the ambiguous case.**
[C, CN, PS, R, T]

Students who have achieved this outcome should be able to:

A. Sketch a diagram to represent a problem that involves a triangle without a right angle.

B. Sketch a diagram and solve a problem, using the cosine law.

C. Sketch a diagram and solve a problem, using the sine law.

D. Describe and explain situations in which a problem may have no solution, one solution or two solutions.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.3 (A C D)

2.4 (A B)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: T3 – Solve problems, using the cosine law and sine law, including the ambiguous case.
[C, CN, PS, R, T]

Elaboration

The measures of the sides and the angles in any triangle can be found by using the sine law and/or the cosine law.

For $\triangle ABC$, the sine law states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

To explain the steps in the sine law proof, draw a height perpendicular to the base in an oblique triangle and manipulate the sine ratios for each of the base angles.

Use the sine law to solve a triangle when you are given the measures of

- two angles and one side;
- two sides and an angle that is opposite to one of the given sides.

Please note that when given two sides and an angle that is opposite to one of the given sides, the ambiguous case may occur when using the sine law.

For $\triangle ABC$, the cosine law states that

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Use the cosine law to solve a triangle when you are given the measures of

- two sides and the angle between them;
- three sides.

To explain the steps in the cosine law proof, draw a height perpendicular to the base in an oblique triangle and manipulate the formula for the cosine ratio for one of the base angles and the formulas for the Pythagorean theorem.

RELATIONS AND FUNCTIONS

SPECIFIC CURRICULUM OUTCOMES

RF1 – Factor polynomials of the form:

- $ax^2 + bx + c$, $a \neq 0$;
- $a^2x^2 - b^2y^2$, $a \neq 0$, $b \neq 0$;

where *a*, *b* and *c* are rational numbers.

RF2 – Graph and analyse absolute value functions (limited to linear and quadratic functions) to solve problems.

RF3 – Analyse quadratic functions of the form $y = a(x-p)^2 + q$ and determine the:

- **vertex;**
- **domain and range;**
- **direction of opening;**
- **axis of symmetry;**
- ***x*- and *y*-intercepts.**

RF4 – Analyse quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including:

- **vertex;**
- **domain and range;**
- **direction of opening;**
- **axis of symmetry;**
- ***x*- and *y*-intercepts**

and to solve problems.

RF5 – Solve problems that involve quadratic equations.

RF6 – Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.

RF7 – Solve problems that involve linear and quadratic inequalities in two variables.

RF8 – Solve problems that involve quadratic inequalities in one variable.

RF9 – Analyse arithmetic sequences and series to solve problems.

RF10 – Analyse geometric sequences and series to solve problems.

RF11 – Graph and analyse reciprocal functions (limited to the reciprocal of linear functions).

MAT521B – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.	**RF1** Factor polynomials of the form: • $ax^2 + bx + c$, $a \neq 0$; • $a^2x^2 - b^2y^2$, $a \neq 0$, $b \neq 0$; where *a*, *b* and *c* are rational numbers.	**RF11** Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree $\leq$ 5 with integral coefficients).

SCO: **RF1 – Factor polynomials of the form:**

- $ax^2 + bx + c$, $a \neq 0$;
- $a^2x^2 - b^2y^2$, $a \neq 0$, $b \neq 0$;

where *a*, *b* and *c* are rational numbers. [CN, ME, R]

Students who have achieved this outcome should be able to:

A. Factor a given polynomial expression that requires the identification of common factors.

B. Factor a given polynomial expression of the form:

- $ax^2 + bx + c$, $a \neq 0$;
- $a^2x^2 - b^2y^2$, $a \neq 0$, $b \neq 0$.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.2 (A B)

4.4 (B)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: RF1 – Factor polynomials of the form:

- $\boldsymbol{ax^2+bx+c}$**,** $\boldsymbol{a \neq 0}$**;**
- $\boldsymbol{a^2x^2-b^2y^2}$**,** $\boldsymbol{a \neq 0, b \neq 0}$**;**

where *a*, *b* and *c* are rational numbers. [CN, ME, R]

Elaboration

Students will extend the polynomial division techniques that were studied in grade nine and the polynomial factoring techniques that were studied in grade ten to this course by factoring polynomials that are more general in format.

In general, each of the following types of polynomials can be factored as follows:

- To factor a polynomial of the form ax^2+bx+c, find two integers with a product of *ac* and having a sum of *b*. Then, expand the middle term as a sum of these two integers. Finally, factor by grouping and removing common factors.

  For example, to factor $2x^2+5x-3$, find two integers with a product of $(2)(-3)$, or –6, and having a sum of 5. Since these integers are –1 and 6, we can expand the trinomial to $2x^2-x+6x-3$. After grouping and removing common factors, we get $x(2x-1)+3(2x-1)$, which factors to $(2x-1)(x+3)$.

- To factor a polynomial of the form $a^2x^2-b^2y^2$, use the fact that $a^2x^2-b^2y^2=(ax+by)(ax-by)$.

  For example, $9x^2-16y^2=(3x+4y)(3x-4y)$.

MAT521B – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
RF4 Describe and represent linear relations, using: • words; • ordered pairs; • tables of values; • graphs; • equations.	**RF2** Graph and analyse absolute value functions (limited to linear and quadratic functions) to solve problems.	

SCO: **RF2 – Graph and analyse absolute value functions (limited to linear and quadratic functions) to solve problems. [C, PS, R, T, V]**

Students who have achieved this outcome should be able to:

A. Create a table of values for $y = \left|f(x)\right|$, given a table of values for $y = f(x)$.

B. Write a rule for absolute value functions in piecewise notation.

C. Sketch the graph of $y = \left|f(x)\right|$, then state the intercepts, domain and range, and explain the strategy used.

D. Solve an absolute value equation graphically, with or without technology.

E. Solve, algebraically, an equation with a single absolute value, and verify the solution.

F. Explain why the absolute value inequality $\left|f(x)\right| < 0$ has no solution.

G. Determine and correct errors in a solution to an absolute value equation.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.2 (A B C F H)

7.3 (D E G H)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF2 – Graph and analyse absolute value functions (limited to linear and quadratic functions) to solve problems. [C, PS, R, T, V]

Elaboration

In general, the absolute value function $y = |f(x)|$ can be written as the piecewise function

$$y = \begin{cases} f(x), & \text{if } f(x) \geq 0 \\ -f(x), & \text{if } f(x) < 0 \end{cases}$$

If $y = f(x)$ is a linear or a quadratic function, then it is easy to determine the domain and range of its corresponding absolute value function, $y = |f(x)|$. The domain of the absolute value function $y = |f(x)|$ is the same as the domain of $y = f(x)$. The range of the absolute value function $y = |f(x)|$ depends on the range of $y = f(x)$. If the graph of $y = f(x)$ crosses or touches the *x*-axis, then the range of $y = |f(x)|$ will be $\{y \mid y \geq 0,\ y \in R\}$. If the graph of $y = f(x)$ does not cross the *x*-axis, then the range of $y = |f(x)|$ will be the same as the range of the function $y = f(x)$.

When solving equations involving absolute value, students should use the piecewise definition of absolute value to help them find the solution(s). As a result, there will always be two cases that have to be considered.

After solving an equation involving absolute value, it is possible that extraneous roots may be introduced. Students must always substitute their possible solutions into the original equation to make sure that they are valid, as there is no easy way to predict whether possible solutions will work in the original equation. For example,

$$|2x+1| = x-2$$

$$2x+1 = x-2 \text{ or } 2x+1 = -(x-2)$$

$$\begin{array}{r|l} 2x - x = -2-1 & 2x+1 = -x+2 \\ x = -3 & 2x+x = 2-1 \\ & 3x = 1 \\ & x = \frac{1}{3} \end{array}$$

Check:

$$|2x+1| = x-2$$

$$\begin{array}{r|l} |2(-3)+1| & -3-2 \\ |-6+1| & -5 \\ |-5| & \\ 5 & \end{array}$$

$$|2x+1| = x-2$$

$$\begin{array}{r|l} \left|2\left(\frac{1}{3}\right)+1\right| & \frac{1}{3}-2 \\ \left|\frac{2}{3}+1\right| & -\frac{5}{3} \\ \left|\frac{5}{3}\right| & \\ \frac{5}{3} & \end{array}$$

Since neither possible solution satisfies the original equation, this equation has no solution.

MAT521B – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
RF5 Determine the characteristics of the graphs of linear relations, including the: • intercepts; • slope; • domain; • range.	**RF3** Analyse quadratic functions of the form $y = a(x-p)^2 + q$ and determine the: • vertex; • domain and range; • direction of opening; • axis of symmetry; • *x*- and *y*-intercepts.	**RF3** Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations. **RF4** Apply translations and stretches to the graphs and equations of functions.

SCO: **RF3 – Analyse quadratic functions of the form $y = a(x-p)^2 + q$ and determine the:**

- **vertex;**
- **domain and range;**
- **direction of opening;**
- **axis of symmetry;**
- ***x*- and *y*-intercepts.**

[CN, R, T, V]

Students who have achieved this outcome should be able to:

A. Explain why a function given in the form $y = a(x-p)^2 + q$ is a quadratic function.

B. Compare the graphs of a set of functions of the form $y = ax^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of *a*.

C. Compare the graphs of a set of functions of the form $y = x^2 + q$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of *q*.

D. Compare the graphs of a set of functions of the form $y = (x-p)^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of *p*.

E. Determine the coordinates of the vertex for a quadratic function of the form $y = a(x-p)^2 + q$, and verify, with or without technology.

F. Generalize, using inductive reasoning, a rule for determining the coordinates of the vertex for quadratic functions of the form $y = a(x-p)^2 + q$.

G. Sketch the graph of $y = a(x-p)^2 + q$, using transformations, and identify the vertex, domain and range, direction of opening, axis of symmetry, and *x*- and *y*-intercepts.

H. Explain, using examples, how the values of *a* and *q* may be used to determine whether a quadratic function has zero, one or two *x*-intercepts.

I. Write a quadratic function of the form $y = a(x-p)^2 + q$ for a given graph or a set of characteristics of a graph.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.1 (A B C D E F G H I)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: RF3 – Analyse quadratic functions of the form $y = a(x-p)^2 + q$ and determine the:

- **vertex;**
- **domain and range;**
- **direction of opening;**
- **axis of symmetry;**
- ***x*- and *y*-intercepts.**

[CN, R, T, V]

Elaboration

For a quadratic function given in vertex form, $f(x) = a(x-p)^2 + q$, where $a \neq 0$, the graph has the following characteristics:

- The shape of the graph is a parabola.
- The vertex of the graph is at the point (p, q).
- The axis of symmetry is the vertical line $x = p$.
- The graph is congruent to $f(x) = ax^2$ translated horizontally by *p* units and vertically by *q* units.
- If $a > 0$, the graph opens upward; if $a < 0$, the graph opens downward.
- If $-1 < a < 1$, where $a \neq 0$, the parabola is wider compared to the graph of $f(x) = x^2$; if $a > 1$ or $a < -1$, the parabola is narrower compared to the graph of $f(x) = x^2$.
- The domain of the graph is the set of all real numbers.
- The range of the graph is $\{y \mid y \geq q,\ y \in R\}$ if $a > 0$, and $\{y \mid y \leq q,\ y \in R\}$ if $a < 0$.
- To determine the number of *x*-intercepts of the graph, use the value of *a* to determine whether the graph opens upward or downward; then use the value of *q* to determine if the vertex is above, below, or on the *x*-axis.

MAT521B – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
RF5 Determine the characteristics of the graphs of linear relations, including the: • intercepts; • slope; • domain; • range.	**RF4** Analyse quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including: • vertex; • domain and range; • direction of opening; • axis of symmetry; • *x*- and *y*-intercepts and to solve problems.	**RF12** Graph and analyse polynomial functions (limited to polynomials functions of degree ≤ 5).

SCO: **RF4 – Analyse quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including:**

- **vertex;**
- **domain and range;**
- **direction of opening;**
- **axis of symmetry;**
- ***x*- and *y*-intercepts**

and to solve problems. **[CN, PS, R, T, V]**

Students who have achieved this outcome should be able to:

A. Convert a quadratic function in standard form to vertex form by completing the square.

B. Write a quadratic function given in the form $y = ax^2 + bx + c$ as a quadratic function in the form $y = a(x-p)^2 + q$.

C. Determine the characteristics of a quadratic function given in the form $y = ax^2 + bx + c$, and explain the strategy used.

D. Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$.

E. Verify, with or without technology, that a quadratic function in the form $y = ax^2 + bx + c$ represents the same function as a given quadratic function in the form $y = a(x-p)^2 + q$.

F. Write a quadratic function that models a given situation, and explain any assumptions made.

G. Solve a problem, with or without technology, by analysing a quadratic function.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.2 (C D F G)

3.3 (A B C)

4.1 (C D G)

4.3 (A B E G)

4.4 (A F G)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: RF4 – Analyse quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including:

- **vertex;**
- **domain and range;**
- **direction of opening;**
- **axis of symmetry;**
- ***x*- and *y*-intercepts**

and to solve problems. [CN, PS, R, T, V]

Elaboration

For a quadratic function given in standard form, $f(x) = ax^2 + bx + c,\ a \neq 0,$ we must rewrite the function by completing the square in order to convert it to vertex form, $f(x) = a(x-p)^2 + q,\ a \neq 0.$ Once the student has completed the square, the characteristics of the quadratic function will be revealed, as shown in Outcome RF3.

The following is the process for completing the square on a quadratic function, $f(x) = ax^2 + bx + c,$ with $a = 1$:

- Group the first two terms of the function.
- Add and subtract the square of one-half times the coefficient of the *x*-term.
- Group the perfect square trinomial.
- Rewrite the perfect square trinomial as the square of a binomial.
- Simplify.

The following is the process for completing the square on a quadratic function, $f(x) = ax^2 + bx + c,$ with $a \neq 1$:

- Take out a common factor from the function by factoring out the coefficient of the x^2-term. Use square brackets to highlight the factored polynomial.
- Within the square brackets, group the first two terms of the function.
- Within the square brackets, add and subtract the square of one-half times the coefficient of the *x*-term.
- Within the square brackets, group the perfect square trinomial.
- Within the square brackets, rewrite the perfect square trinomial as the square of a binomial.
- Within the square brackets, simplify.
- Remove the square brackets by placing the common factor in front of the parentheses and multiplying it by the constant that was within the square bracket.

MAT521B – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.	**RF5** Solve problems that involve quadratic equations.	

SCO: **RF5 – Solve problems that involve quadratic equations.** **[C, CN, PS, R, T, V]**

Students who have achieved this outcome should be able to:

A. Explain, using examples, the relationships among the roots of a quadratic equation, the zeros of its corresponding quadratic function and the *x*-intercepts of the graph of the quadratic function.

B. Solve a quadratic equation of the form $ax^2 + bx + c = 0$ by using strategies such as:
- determining square roots;
- factoring;
- applying the quadratic formula;
- graphing its corresponding function.

C. Select a method for solving a quadratic equation, justify the choice, and verify the solution.

D. Identify and correct errors in a solution to a quadratic equation.

E. Solve a problem by:
- analysing a quadratic equation;
- determining and analysing a quadratic equation.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.1 (A B C D E)

4.2 (B C D E)

4.3 (B C D E)

4.4 (B C D E)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: RF5 – Solve problems that involve quadratic equations. **[C, CN, PS, R, T, V]**

Elaboration

Depending on the type of quadratic equation, there are a number of strategies that can be used to solve it.

If the equation is in vertex form, $a(x-p)^2+q=0$, the equation can be solved by applying the square root principle.

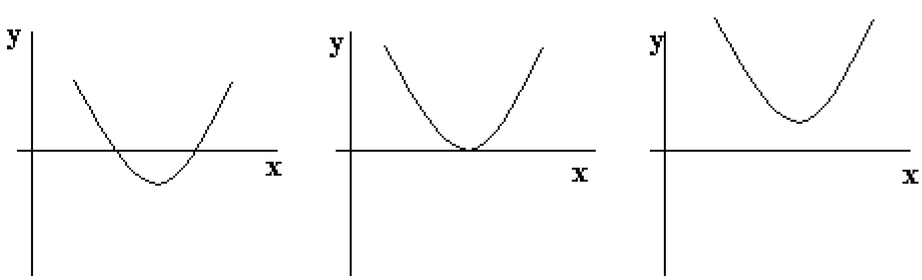
If ax^2+bx+c is factorable, then to solve $ax^2+bx+c=0$, factor the quadratic expression and rewrite it in factored form. Then, using the zero product property, set each factor equal to zero and solve.

If ax^2+bx+c is not factorable, then the equation $ax^2+bx+c=0$ can be solved by using the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, or by graphing the function $y=ax^2+bx+c$, and determining where the zeros of the function lie. Please note however, that graphing will only produce approximate answers in many cases. To derive the formula for the quadratic formula, the process of completing the square is used.

The expression b^2-4ac, known as the discriminant, can be used when applying the quadratic formula to determine the nature of the roots of the quadratic equation $ax^2+bx+c=0$.

- When $b^2-4ac>0$, there are two distinct real roots. The graph of the corresponding function has two different *x*-intercepts.
- When $b^2-4ac=0$, there is one distinct real root, or two equal real roots. The graph of the corresponding function has one *x*-intercept.
- When $b^2-4ac<0$, there are no real roots. The graph of the corresponding function has no *x*-intercepts.

$b^2-4ac>0$ $\qquad$ $b^2-4ac=0$ $\qquad$ $b^2-4ac<0$



MAT521B – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.	**RF6** Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.	

SCO: **RF6 – Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.** [CN, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations graphically, with technology.

B. Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations algebraically.

C. Explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations.

D. Explain, using examples, why a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two, or an infinite number of solutions.

E. Solve a problem that involves a system of linear-quadratic or quadratic-quadratic equations, and explain the strategy used.

Note: It is intended that the quadratic equations be limited to those that correspond to quadratic functions.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

8.1 (A C D E)

8.2 (B B)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: RF6 – Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables. [CN, PS, R, T, V]

Elaboration

Systems of linear-quadratic equations may have no solution, one solution, or two solutions. Systems of quadratic-quadratic equations may have no solution, one solution, two solutions, or an infinite number of solutions. Such systems of equations may be solved algebraically by using either a substitution or an elimination method.

To solve a system of equations in two variables using substitution,

- isolate one variable in one equation, preferably one that is not squared,
- substitute the expression into the other equation and solve for the remaining variable,
- substitute the value(s) into one of the original equations to determine the corresponding value(s) of the other variable;
- verify the answer(s) by substituting into both original equations.

To solve a system of equations in two variables using elimination,

- if necessary, rearrange the equations so that the like terms align;
- if necessary, multiply one or both equations by a constant to create equivalent equations with a pair of variable terms with opposite coefficients;
- add or subtract to eliminate one variable and solve for the remaining variable;
- substitute the value(s) into one of the original equations to determine the corresponding value(s) of the other variable;
- verify the answer(s) by substituting into both original equations.

MAT521B – Topic: Relations and Functions (RF)
GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
RF4 Describe and represent linear relations, using: • words; • ordered pairs; • tables of values; • graphs; • equations.	**RF7** Solve problems that involve linear and quadratic inequalities in two variables.	

SCO: RF7 – Solve problems that involve linear and quadratic inequalities in two variables. [C, PS, T, V]

Students who have achieved this outcome should be able to:

A. Explain, using examples, how test points can be used to determine the solution region that satisfies an inequality.

B. Explain, using examples, when a solid or broken line should be used in the solution for an inequality.

C. Sketch, with or without technology, the graph of a linear or quadratic inequality.

D. Solve a problem that involves a linear or quadratic inequality.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

9.1 (A B C D)

9.3 (C D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF7 – Solve problems that involve linear and quadratic inequalities in two variables. [C, PS, T, V]

Elaboration

The boundary of the solution region of an inequality is the graph of the related linear or quadratic equation.

- When the inequality symbol is $\leq$ or $\geq$, the points on the boundary are included in the solution region, and the boundary is a solid line or parabola.
- When the inequality symbol is $<$ or $>$, the points on the boundary are not included in the solution region, and the boundary is a dashed line or parabola.

To determine which part of the Cartesian plane is shaded when graphing an inequality, select a test point on one side of the boundary.

- If the test point satisfies the original inequality, then that side of the boundary is shaded.
- If the test point does not satisfy the original inequality, then the other side of the boundary is shaded.

Please note that as long as it does not lie on the boundary, $(0,0)$ serves as a good test point, as it is easy to evaluate.

An alternate method used to determine which half of the graph to shade is to solve the original inequality for *y*, keeping the *y* on the left-hand side of the inequality. If the resulting inequality includes a greater than sign $(>$ or $\geq)$, then the region above the boundary is shaded. If the resulting inequality includes a less than sign $(<$ or $\leq)$, then the region below the boundary is shaded. Please note that if the boundary is a vertical line, an inequality including a less than sign indicates to shade the left side of the boundary line, and an inequality including a greater than sign indicates to shade the right side of the boundary line.

MAT521B – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
	RF8 Solve problems that involve quadratic inequalities in one variable.	

SCO: **RF8 – Solve problems that involve quadratic inequalities in one variable. [CN, PS, V]**

Students who have achieved this outcome should be able to:

A. Determine the solution of a quadratic inequality in one variable.

B. Represent and solve a problem that involves a quadratic inequality in one variable.

C. Interpret the solution to a problem that involves a quadratic inequality in one variable.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

9.2 (A B C)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: RF8 – Solve problems that involve quadratic inequalities in one variable. [CN, PS, V]

Elaboration

The solution to a quadratic inequality in one variable is a set of values. Before solving a quadratic inequality, it is best to re-arrange it so that there is a zero on the right-hand side. There are a number of strategies that can be used to solve the resulting inequality.

- Graph the corresponding function, and identify the values of *x* for which the function lies on, above, or below the *x*-axis, depending on the inequality symbol.
- Determine the roots of the related equation, and then use a number line and test points to determine the intervals that satisfy the inequality.
- Determine when each of the factors of the quadratic expression is positive, zero, or negative, and then use the results to determine the sign of the product in each case.
- Consider all cases for the required product of the factors of the quadratic expression to find any *x*-values that satisfy both factor conditions in each case.

Remember that if the original quadratic inequality contains an equal sign, then the solutions will also contain an equal sign.

MAT521B – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
	RF9 Analyse arithmetic sequences and series to solve problems.	

SCO: **RF9 – Analyse arithmetic sequences and series to solve problems. [CN, PS, R, T]**

Students who have achieved this outcome should be able to:

A. Identify the assumption(s) made when defining an arithmetic sequence or series.

B. Provide and justify an example of an arithmetic sequence.

C. Determine the general term of an arithmetic sequence.

D. Determine t_1, d, n, or t_n in a problem that involves an arithmetic sequence.

E. Determine a rule for the sum of n terms of an arithmetic series.

F. Determine t_1, d, n, or S_n in a problem that involves an arithmetic series.

G. Solve a problem that involves an arithmetic sequence or series.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (A B C D G)

1.2 (A E F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: RF9 – Analyse arithmetic sequences and series to solve problems. [CN, PS, R, T]

Elaboration

In an arithmetic sequence, each successive term is determined by adding a constant, *d*, called the common difference, to the previous term. An arithmetic series is the sum of the terms of an arithmetic sequence.

The general term of an arithmetic sequence is $t_n = t_1 + (n-1)d$, where t_1 is the first term, *n* (a natural number) is the number of terms, *d* is the common difference, and t_n is the general, or *n*th term.

The sum of the first *n* terms of an arithmetic series can be found using $S_n = \frac{n}{2}\left[2t_1 + (n-1)d\right]$ or $S_n = \frac{n}{2}(t_1 + t_n)$, where t_1 is the first term, *n* (a natural number) is the number of terms, *d* is the common difference, t_n is the general, or *n*th term, and S_n is the sum of the first *n* terms.

The second formula for S_n can be derived from the first formula, as follows:

$$\begin{aligned} S_n &= \frac{n}{2}\left[2t_1 + (n-1)d\right] \\ &= \frac{n}{2}\left[t_1 + t_1 + (n-1)d\right] \\ &= \frac{n}{2}(t_1 + t_n), \qquad \text{since } t_n = t_1 + (n-1)d \end{aligned}$$

MAT521B – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
	RF10 Analyse geometric sequences and series to solve problems.	

SCO: **RF10 – Analyse geometric sequences and series to solve problems. [PS, R, T]**

Students who have achieved this outcome should be able to:

A. Identify assumptions made when identifying a geometric sequence or series.

B. Provide and justify an example of a geometric sequence.

C. Determine the general term of a geometric sequence.

D. Determine t_1, r, n, or t_n in a problem that involves a geometric sequence.

E. Determine a rule for the sum of n terms of a geometric series.

F. Determine t_1, r, n, or S_n in a problem that involves a geometric series.

G. Explain why a geometric series is convergent or divergent.

H. Solve a problem that involves a geometric sequence or series.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.3 (A B C D H)

1.4 (A E F H)

1.5 (G H)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: RF10 – Analyse geometric sequences and series to solve problems. [PS, R, T]

Elaboration

In a geometric sequence, each successive term is determined by multiplying a constant, *r*, called the common ratio, to the previous term. A geometric series is the sum of the terms of a geometric sequence.

The general term of a geometric sequence is $t_n = t_1 r^{n-1}$, where t_1 is the first term, *n* (a natural number) is the number of terms, *r* is the common ratio, and t_n is the general, or *n*th term.

The sum of the first *n* terms of a geometric series is $S_n = \frac{t_1(r^n - 1)}{r-1}$, $r \neq 1$ or $S_n = \frac{rt_n - t_1}{r-1}$, $r \neq 1$ (if *n* is unknown), where t_1 is the first term, *n* (a natural number) is the number of terms, *r* is the common ratio, t_n is the general, or *n*th term, and S_n is the sum of the first *n* terms.

To derive the second formula from the first formula, begin with the formula for the general term of a geometric sequence, $t_n = t_1 r^{n-1}$. Multiplying both sides by *r* gives us $rt_n = (t_1 r^{n-1})(r)$, which simplifies to $rt_n = t_1 r^n$. If we multiply out the first formula for a geometric series, we get, $S_n = \frac{t_1 r^n - t_1}{r-1}$. Applying the substitution $rt_n = t_1 r^n$ gives us the final result $S_n = \frac{rt_n - t_1}{r-1}$.

The sum of an infinite geometric series, where $-1 < r < 1$, can be determined using the formula $S_\infty = \frac{t_1}{1-r}$, where S_∞ is the sum of the infinite series, t_1 is the first term of the series, and *r* is the common ratio. To derive the formula for an infinite geometric series, use inductive reasoning and the formula for a finite geometric series.

MAT521B – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 10 – MAT421A	GRADE 11 – MAT521B	GRADE 12 – MAT621B
RF4 Describe and represent linear relations, using: • words; • ordered pairs; • tables of values; • graphs; • equations.	**RF11** Graph and analyse reciprocal functions (limited to the reciprocal of linear functions).	**RF5** Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the: • *x*-axis; • *y*-axis; • line $y = x$. **RF6** Demonstrate an understanding of inverses of relations.

SCO: **RF11 – Graph and analyse reciprocal functions (limited to the reciprocal of linear functions).**
[CN, R, T, V]

Students who have achieved this outcome should be able to:

A. Compare the graph of $y = \frac{1}{f(x)}$ to the graph of $y = f(x)$.

B. Identify, given a function $y = f(x)$, values of *x* for which $y = \frac{1}{f(x)}$ will have vertical asymptotes; and describe their relationship to the non-permissible values of the related rational expression.

C. Graph, with or without technology, $y = \frac{1}{f(x)}$, given $y = f(x)$ as a function or a graph, and explain the strategies used.

D. Graph, with or without technology, $y = f(x)$, given $y = \frac{1}{f(x)}$ as a function or a graph, and explain the strategies used.

Section(s) in Pre-Calculus 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.4 (A B C D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF11 – Graph and analyse reciprocal functions (limited to the reciprocal of linear functions).
[CN, R, T, V]

Elaboration

The graph of $y = \frac{1}{f(x)}$ can be obtained from the graph of $y = f(x)$ by using the following guidelines:

- The non-permissible values of the reciprocal function $y = \frac{1}{f(x)}$ are the same as the zeros of the function $y = f(x)$.
- The non-permissible values of the reciprocal function are related to the position of the vertical asymptotes of its graph. These are also the non-permissible values of the corresponding rational expression, or where the reciprocal function is undefined.
- If the graph of $y = f(x)$ is increasing, then the graph of $y = \frac{1}{f(x)}$ is decreasing.
- If the graph of $y = f(x)$ is decreasing, then the graph of $y = \frac{1}{f(x)}$ is increasing.
- Invariant points occur when the function $f(x)$ has a value of 1 or –1. To determine the *x*-coordinates of the invariant points, solve the equations $f(x) = \pm 1$.
- The *y*-coordinates of the points on the graph of the reciprocal function are the reciprocals of the *y*-coordinates of the corresponding points on the graph of $y = f(x)$.
- As the value of *x* approaches a non-permissible value, the absolute value of the reciprocal function gets very large.
- As the absolute value of *x* gets very large, the absolute value of the reciprocal function approaches zero.

REFERENCES

American Association for the Advancement of Science [AAAS-Benchmarks]. *Benchmark for Science Literacy*. New York, NY: Oxford University Press, 1993.

Banks, James A. and Cherry A. McGee Banks. *Multicultural Education: Issues and Perspectives*. Boston: Allyn and Bacon, 1993.

Black, Paul and Dylan Wiliam. "Inside the Black Box: Raising Standards Through Classroom Assessment." *Phi Delta Kappan*, 20, October 1998, pp.139-148.

Canavan-McGrath, Cathy, *et. al. Foundations of Mathematics 11*. Nelson Education, 2012.

British Columbia Ministry of Education. *The Primary Program: A Framework for Teaching*, 2000.

Davies, Anne. *Making Classroom Assessment Work*. British Columbia: Classroom Connections International, Inc., 2000.

Hope, Jack A., *et. al*. *Mental Math in the Primary Grades*. Dale Seymour Publications, 1988.

National Council of Teachers of Mathematics. *Mathematics Assessment: A Practical Handbook*. Reston, VA: NCTM, 2001.

National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

Rubenstein, Rheta N. *Mental Mathematics Beyond the Middle School: Why? What? How?* September 2001, Vol. 94, Issue 6, p. 442.

Shaw, Jean M. and Mary Jo Puckett Cliatt. "Developing Measurement Sense." In P.R. Trafton (ed.), *New Directions for Elementary School Mathematics* (pp. 149–155). Reston, VA: NCTM, 1989.

Steen, Lynn Arthur (ed.). *On the Shoulders of Giants – New Approaches to Numeracy.* Washington, DC: National Research Council, 1990.

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades 5-8.* Boston: Pearson Education, Inc. 2006.

Western and Northern Canadian Protocol. *Common Curriculum Framework for 10-12 Mathematics*, 2008.