Prince Edward Island
CANADA
Education and Early Years

Prince Edward Island Mathematics Curriculum

Mathematics

CURRICULUM

Grade 6

Acknowledgments

The Department of Education and Early Years of Prince Edward Island gratefully acknowledges the contributions of the following groups and individuals toward the development of the *Prince Edward Island Grade 6 Mathematics Curriculum Guide*:

- Eamon Graham
  Elementary Mathematics & Science Specialist, French Curriculum, Department of Education, Early Learning and Culture

  J. Blaine Bernard
  Secondary Mathematics Specialist, English Curriculum, Department of Education, Early Learning and Culture

  Ted Johnston
  Elementary Mathematics & Science Specialist, English Curriculum, Department of Education, Early Learning and Culture

  Brenda Larsen
  Elementary Mathematics & Science Specialist English Curriculum, Department of Education, Early Learning and Culture

  Lauren Gill,
  K-9 Mathematics Curriculum Leader,
  Department of Education and Early Years

- The 2010-2011 grade six mathematics pilot teachers:

  Cheryl Turner, Athena Consolidated
  David Delgado, Greenfield Elementary
  Stephanie Rooney, Englewood School
  Sheryl O'Hanley, Morell Consolidated
  Nadine Gallant, Tignish Elementary
  Linda Inman - Hyson, West Royalty Elementary
  Patti McQuaid, Glen Stewart Elementary
  Ellen Read, West Royalty Elementary
  Lindsay Baillie, Greenfield Elementary
  Lisa Paynter, Parkside Elementary
  Paul Sullivan, Southern Kings Consolidated
  Elsie Moore, Southern Kings Consolidated
  Jill Richard, Athena Consolidated
  Linda Trainor, Stratford Elementary School
  Carol Clarke, Amherst Cove Consolidated
  Elaine Murphy, Stratford Elementary School
  Dawn Ward, Stratford Elementary School
  Maureen MacBeath, West Royalty Elementary
  Marie McMahon - Young, Stratford Elementary School

- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education
- The Alberta Department of Education
- The New Brunswick Department of Education

Prince Edward Island
Department of Education and Early Years
Holman Centre
250 Water Street, Suite 101
Summerside, Prince Edward Island
Canada C1N 1B6
Implemented 2011
Revised 2023

Table of Contents

Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base in its creation. From examining the curriculum proposed throughout Canada to securing the latest research in the teaching of mathematics, the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work and study today and in the future. Essential graduation learnings are cross-curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will be able to demonstrate knowledge, skills and attitudes in the following essential graduation learnings:

- Respond with critical awareness to various forms of the arts and be able to express themselves through the arts.
- Assess social, cultural, economic and environmental interdependence in a local and global context.
- Use the listening, viewing, speaking and writing modes of language(s), and mathematical and scientific concepts and symbols to think, learn and communicate effectively.
- Continue to learn and to pursue an active, healthy lifestyle.
- Use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts.
- Use a variety of technologies, demonstrate an understanding of technological applications and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate high expectations for students in mathematics education to all educational partners. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning; and
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

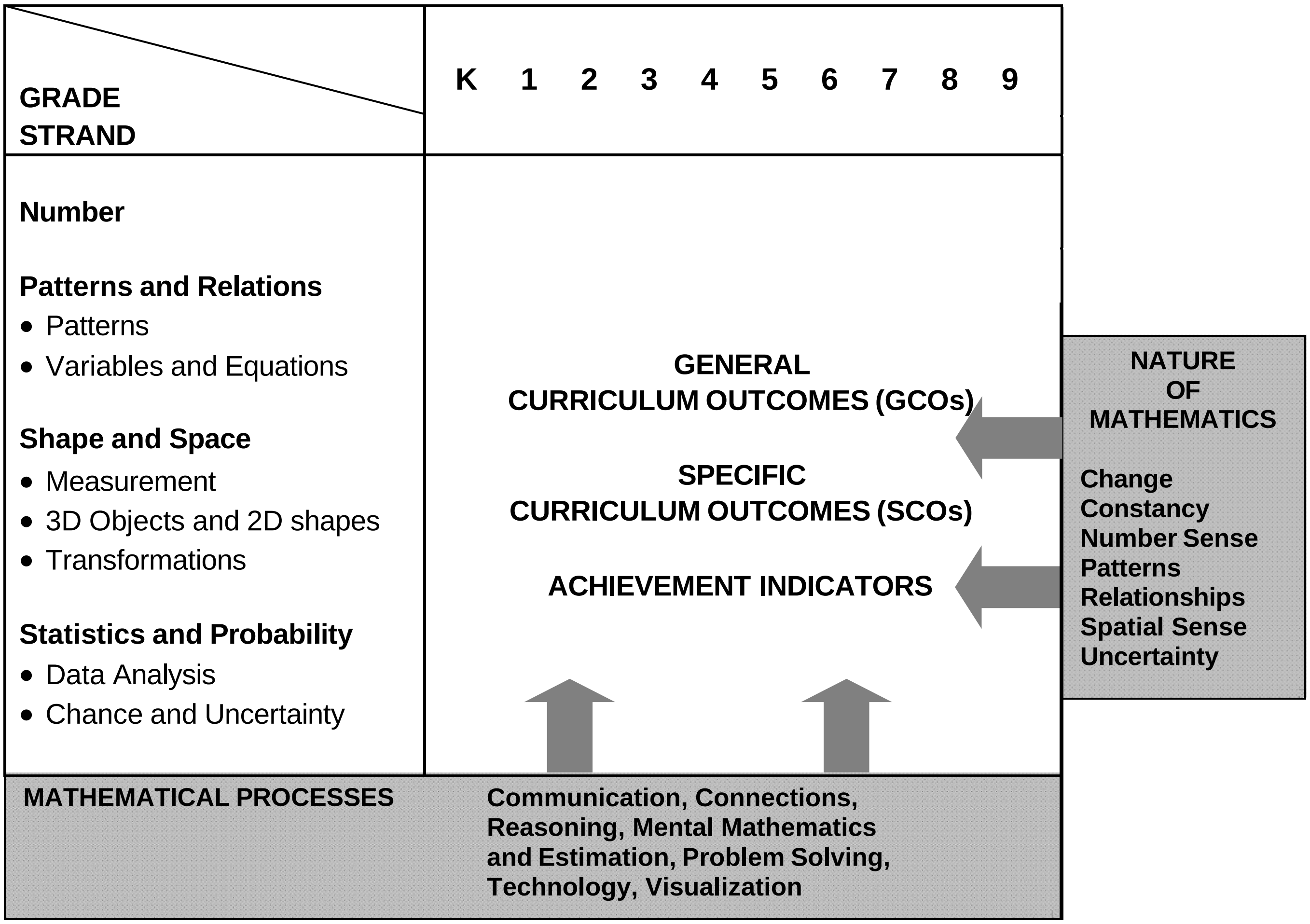
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks; and
- exhibit curiosity.

➢ 21st Century Learning

According to Bernie Trilling and Charles Fadel in *21st Century Skills* (2009), “critical thinking and problem solving are considered ... the new basics of 21st century learning”. They further state “...using knowledge as it is being learned – applying skills like critical thinking, problem solving, and creativity to the content knowledge – increases motivation and improves learning outcomes.” (Trilling & Fadel, 2009).

Conceptual Framework for K – 9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes:



The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely: **Number**, **Patterns and Relations**, **Shape and Space**, and **Statistics and Probability**. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.

➢ There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to:

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]** and
- develop visualization skills to assist in processing information, making connections and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

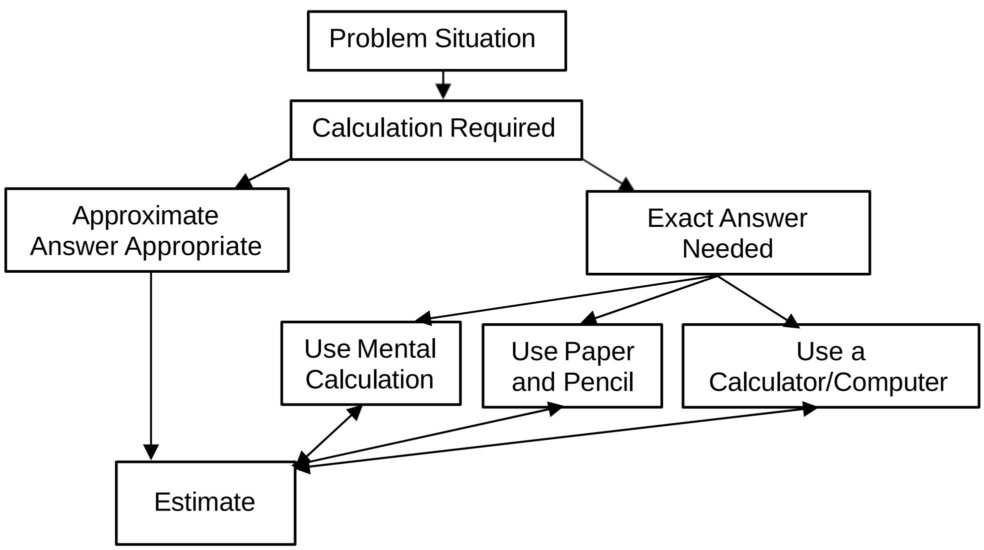
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001). Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision making process as described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you. . . ?" or "How could you. . . ?" the problem-solving approach is being modeled. Students develop their own problem-solving strategies by being open to listening, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modeled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- use estimation
- guess and check
- look for a pattern
- make an organized list or table
- use a model
- work backwards
- use a formula
- use a graph, diagram or flow chart
- solve a simpler problem
- use algebra

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations; and
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K–3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3D objects and 2D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, to determine when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change**, **constancy**, **number sense**, **patterns**, **relationships**, **spatial sense** and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution;
- the sum of the interior angles of any triangle is 180°; and
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3D and 2D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3D objects and 2D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3D or 2D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations;
- the volume of a rectangular solid can be calculated from given dimensions; and
- doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking and critical thinking and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Connections across the Curriculum

There are many possibilities for connecting Grade 6 mathematical learning with the learning occurring in other subject areas. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learning. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should reduce some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a parent will have a clearer understanding of the mathematics curriculum and the progress of his or her child in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every classroom comprises students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters but should be designed to help all students, whether strong, weak or average, to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson, from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean that not only should enrolments of students of both genders and various cultural backgrounds in public school mathematics courses reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English proficiency and cultural differences must not be a barrier to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and coordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education (p.60)." The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers that will facilitate "communicating to learn mathematics and learning to communicate mathematically (NCTM, p.60)."

To this end:

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counselors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated, with appropriate language support, to both students and parents; and
- to verify that barriers have been removed, educators should monitor enrollment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development – such as poverty alleviation, human rights, health, environmental protection and climate change – into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental and economic perspective and explores how those factors are inter-related and inter-dependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database

Resources for Rethinking, found at http://r4r.ca/en. It provides teachers with access to materials that integrate ecological, social and economic spheres through active, relevant, interdisciplinary learning.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, whether teaching has been effective or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated and how results are communicated send clear messages to students and others.

➢ Assessment

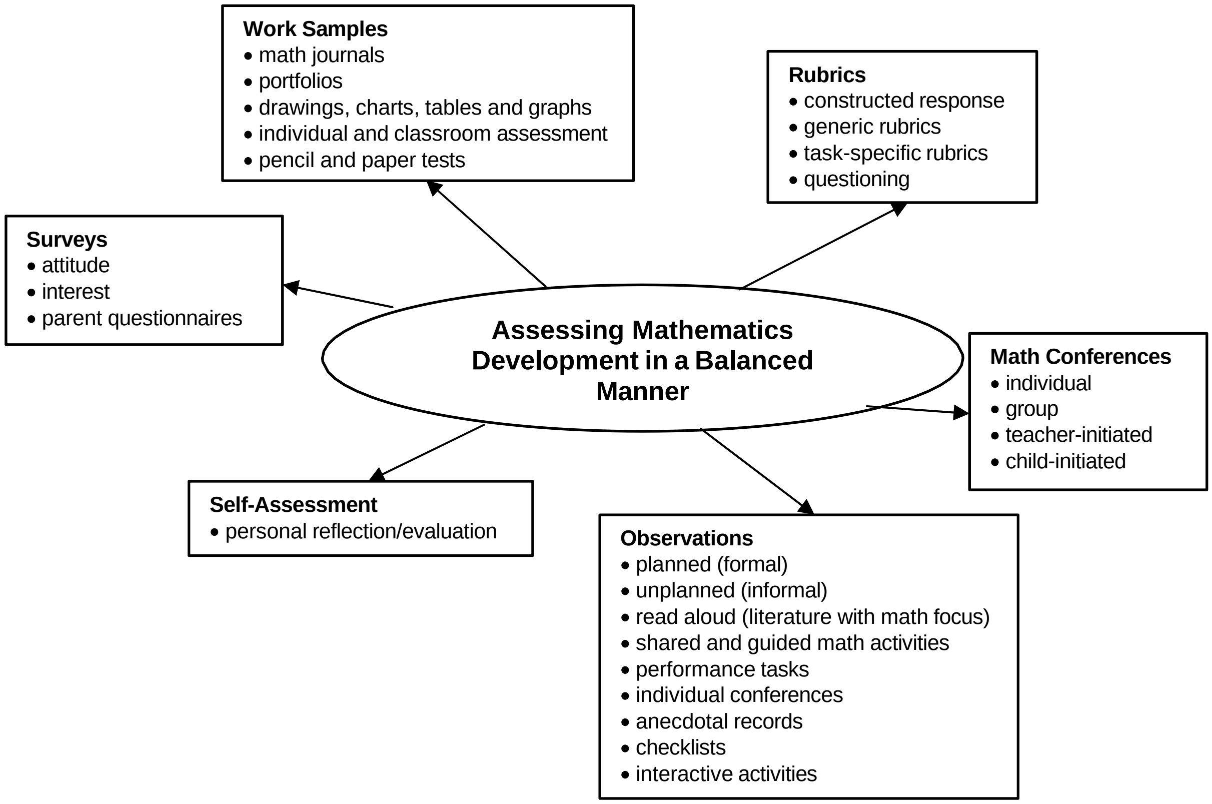
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as:

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources including:

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment

This balanced approach for assessing mathematics development is illustrated in the diagram below.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used:

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used:

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used:

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;
- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon and summarizing assessment information and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires:

- student learning;
- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information; and
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes and phone calls.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.

- Assessment reports should be clear, accurate and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that:

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes; and
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

Provincial Assessment Program

Island students participate in provincial, national and international assessments that help measure individual and overall student achievement and the overall performance of our provincial education system.

Provincial assessments are conducted yearly and tell us how well students are doing at key stages of learning. Students are assessed in reading, writing and mathematics at the end of Grade 3, Grade 6, and Grade 9.

These provincial assessments are developed by teachers from across the province and are based on the curriculum used in Island schools. These assessments tell us how well students are learning the curriculum, where students may need help, and how resources may be directed to support students attaining a deeper understanding of mathematical thinking.

Provincial assessments are just one of many tools used to monitor student learning. Parents should talk to the teacher about the full scope of their child's performance. Working together with good information, parents and teachers can help students to reach their full potential.

The Department of Education and Early Childhood Development also supports a national assessment which takes place every three years. The Pan-Canadian Assessment Program (PCAP) assesses the performance of 13-year-old students in reading, math and science.

Every three years, Island students also take part in the Programme for International Student Assessment (PISA), which assesses the achievement of 15-year-old students in reading, math and science through a common international test.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are **Number, Patterns and Relations, Shape and Space**, and **Statistics and Probability**. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense.
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems.
	Variables and Equations: Represent algebraic expressions in multiple ways.
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems.
	3D Objects and 2D Shapes: Describe the characteristics of 3D objects and 2D shapes, and analyze the relationships among them.
	Transformations: Describe and analyze position and motion of objects and shapes.
Statistics and Probability (SP)	**Data Analysis**: Collect, display, and analyze data to solve problems.
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

The first two pages for each outcome contain the following information:

- the corresponding **strand** and **General Curriculum Outcome**;
- the **Specific Curriculum Outcome**(s) and the mathematical **processes** which link this content to instructional methodology;
- the **scope and sequence** of concept development related to this outcome(s) from grades 5 - 7;
- a list of **achievement indicators**; and

  Students who have achieved a particular outcome should be able to demonstrate their understanding in the manner specified by the achievement indicators. It is important to remember, however, that these indicators are not intended to be an exhaustive list for each outcome. Teachers may choose to use additional indicators as evidence that the desired learning has been achieved.
- an **elaboration** of the outcome.

The last two pages for each outcome contain lists of **instructional strategies** and **strategies for assessment**.

The primary use of this section of the guide is as an ***assessment for learning*** (formative assessment) tool to assist teachers in planning instruction to improve learning. However, teachers may also find the ideas and suggestions useful in gathering ***assessment of learning*** (summative assessment) data to provide information on student achievement.

The **Mental Math Guide,** which outlines the **Fact Learning, Mental Computation and Estimation** strategies for this grade level, can be found at learn.edu.pe.ca. Included is an **Overview of the Thinking Strategies in Mental Math** for grades one to six complete with a description of each strategy as well as a scope and sequence table of the strategies for the elementary grades.

NUMBER

SPECIFIC CURRICULUM OUTCOMES

6.N1 – Demonstrate an understanding of place value for numbers: greater than one million; less than one thousandth.

6.N2 – Demonstrate an understanding of factors and multiples by: determining multiples and factors of numbers less than 100 identifying prime and composite numbers; solving problems involving multiples.

6.N3 – Relate improper fractions to mixed numbers.

6.N4 – Demonstrate an understanding of percent, (limited to whole numbers) concretely, pictorially and symbolically.

6.N5 – Demonstrate an understanding of integers, concretely, pictorially and symbolically.

6.N6 – Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).

6.N7 – Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).

SCO: **6.N1 Demonstrate an understanding of place value for numbers:**
- **greater than one million**
- **less than one thousandth.**

[C, CN, R, T]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.N1 Represent and describe whole numbers to 1 000 000. **5.N8** Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.	**6.N1** Demonstrate an understanding of place value for numbers: • greater than one million • less than one thousandth.	**7.N2** Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems. **7.N6** Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using: • benchmarks; • place value; • equivalent fractions and/or decimals.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. explain how the pattern of the place value system, e.g., the repetition of ones, tens and hundreds, makes it possible to read and write numerals for numbers of any magnitude; and

B. provide examples of where large numbers and small decimals are used, e.g., media, science, medicine, technology.

SCO: **6.N1 Demonstrate an understanding of place value for numbers:**
- **greater than one million**
- **less than one thousandth.**

[C, CN, R, T]

Elaboration

Students will extend their knowledge from numbers in the millions by discovering patterns that go beyond to the billions and trillions. Students should understand that the place value system follows a pattern such that:

- each position represents ten times as much as the position to its right;
- each position represents one tenth as much as the position to its left;
- positions are grouped in threes for purposes of reading numbers;
- when writing numbers, spaces (not commas) are used to show the positions with the exception of 4-digit numbers (e.g., 5640).

All students should be aware that numbers extend to the left up to infinity, and to the right into the ten thousandths, hundred thousandths and millionths places, and so on.
Students should have many opportunities to:

- read numbers several ways: for example, 6732.14 could be read as six thousand, seven hundred thirty-two and fourteen hundredths or sixty-seven hundred, thirty-two and fourteen hundredths;
- read numbers greater than a thousand: 2 456 870 346 is read two billion, four hundred fifty-six million, eight hundred seventy thousand, three hundred forty-six ("and" is used for decimal numbers);
- record numbers: for example, ask students to write twelve million, one hundred thousand in **standard form** (12 100 000) and **decimal notation** (12.1 million) (scientific notation will be explored in later grades);
- establish **personal referents** to develop a sense of larger numbers (e.g., local arena holds 500 people, population of their town is 10 000, a school/class collection of over a million small objects).

Through these experiences, students will develop flexibility in identifying and representing numbers beyond 1 000 000. It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real life contexts that are personally meaningful (e.g., computer memory size, professional athletes' salaries, Internet search responses, populations, or the microscopic world).

Students also need to know that the place value system extends to the right as well and that there are numbers smaller than 0.001.

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:

- Unit 2, Lesson 1, pp. 46-50
- Unit 3, Lesson 1, pp. 88-91

SCO: **6.N1 Demonstrate an understanding of place value for numbers:**
- **greater than one million**
- **less than one thousandth.**

[C, CN, R, T]

Instructional Strategies

Consider the following strategies when planning lessons:

- Ask students to find various representations for multi-digit and decimal numbers in newspapers and magazines. Encourage discussion on the need for accuracy in reporting these numbers and the appropriate use of rounded numbers.
- Present a meter stick as a number line from zero to one billion. Ask students where one million, half a billion, one hundred million, etc., would be on this number line.
- Write decimals using place value language and expanded notation to help explain equivalence of decimals.
  0.2 = 2 tenths
  0.20 = 2 tenths + 0 hundredths
  0.200 = 2 tenths + 0 hundredths + 0 thousandths
  } Since adding zeros have no effect, 0.2 must equal 0.20 and 0.200
- Ensure that proper vocabulary is used when reading all numbers. Provide opportunities for students to read decimals in context. Saying decimals correctly will help students make the connection between decimals and fractions 5.0072 should be read as "five and seventy-two ten thousandths" not "five point zero, zero, seven, two".

Suggested Activities

- Ask students to create an "A-B-C" book that includes real-world examples of very large numbers and very small decimals (e.g., population of Mexico City; length of an ant's antenna in centimetres).
- Prepare and shuffle 5 sets of number cards (0-9 for each set). Have the students select nine cards and ask them to arrange the cards to make the greatest possible and least possible whole number. Have the students read each of the numbers. Consider extending the activity by asking students to determine:
  - how many different whole numbers could be made using the nine digits selected;
  - the number of $1000 bills one would get if the greatest and least numbers represented money amounts. This could be extended to explore the number of tens, hundreds, etc. in the number.
- Discuss words people use for large number that do not exist, i.e., gazillion, bazillion. This might peak students' interest to research other prefixes past trillions.
- Ask students to determine the number of whole numbers between 2.03 million and 2.35 million.
- Ask students to find a value between 0.0001 and 0.00016.
- Include contexts that lend themselves to using large numbers such as astronomical data and demographic data. Contexts that lend themselves to decimal thousandths include sports data and metric measurements. An interesting activity involving decimals might require students to complete a chart such as: in 0.1 years, I could...; in 0.01 years, I could...; in 0.001 years, I could...
- Present this library information to students: Metropolitan Toronto Library 3 068 078 books; Bibliothèque de Montreal 2 911 764 books; North York Public Library 2 431 655 books. Ask students to rewrite the numbers in a format such as ☐ .☐million or ☐ .☐☐million books. Then ask them to make comparison statements about the number of books.

SCO: **6.N1 Demonstrate an understanding of place value for numbers:**
- **greater than one million**
- **less than one thousandth.**

[C, CN, R, T]

Assessment Strategies

- Have students explain at least three things they know about a number with 10 digits.
- Ask students to describe when 1 000 000 000 of something might be a big amount? A small amount?
- Have students generate a number with 7 – 10 digits. Have them find classmates with numbers that are similar (place value). Once they have found a group they belong to, have them order their numbers from least to greatest. Then have the class order the numbers from least to greatest. Have each student read their number. (This activity can be done in silence so students have to really look at the other numbers). This activity can be done using decimal numbers as well.
- Ask students to express 0.00674 in at least three different ways.
- Ask students to describe how the bolded digits in the following two numbers are the same and how they are different.

  5**4**6 397 305 3**4**8 167 903 927

  Extend the activity to decimals:

  0.00**7**0 0.000**7**
- Ask students to write a report on what he/she has learned about decimals and what questions he/she may now have concerning the topic.

SCO: **6.N2 Demonstrate an understanding of factors and multiples by:**
- **determining multiples and factors of numbers less than 100**
- **identifying prime and composite numbers**
- **solving problems involving multiples.**

[PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.N3 Apply mental mathematics strategies and number properties, such as: skip counting from a known fact; using doubling or halving; using patterns in the 9s facts; using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts.	**6.N2** Demonstrate an understanding of factors and multiples by: • determining multiples and factors of numbers less than 100 • identifying prime and composite numbers • solving problems involving multiples.	**7.N1** Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. identify multiples for a given number and explain the strategy used to identify them;

B. determine all the whole number factors of a given number using arrays;

C. identify the factors for a given number and explain the strategy used, e.g., concrete or visual representations, repeated division by prime numbers or factor trees;

D. provide an example of a prime number and explain why it is a prime number;

E. provide an example of a composite number and explain why it is a composite number;

F. sort a given set of numbers as prime and composite and;

G. solve a given problem involving factors or multiples.

SCO: **6.N2 Demonstrate an understanding of factors and multiples by:**
- **determining multiples and factors of numbers less than 100**
- **identifying prime and composite numbers**
- **solving problems involving multiples.**

[PS, R, V]

Elaboration

Multiples of a whole number are the products of that number and any other whole number. To find the first four multiples of 3, multiply 3 by 1, 2, 3, and 4 to get the multiples 3, 6, 9, 12. Multiples of a number can also be found by skip counting by that number.

Factors are numbers that are multiplied to get a **product** (3 and 4 are factors of 12). The factors for a number can be found by dividing the number by smaller numbers and looking to see if there is a remainder of zero. At this point students should also recognize that:
- the factors of a number are never greater than the number;
- the greatest factor is always the number itself; the least factor is one;
- the second factor is always half the number or less;
- the multiple of a number always has that number as a factor.

To help students understand the meanings for the terms "factor" and "multiple" students could explore these concepts and write their own definition (e.g., *factor* × *factor* = *multiple*).

A **prime** number is defined as a number which has only 2 factors: 1 and itself (e.g., 29 only has factors of 1 and 29 is therefore prime). Students should recognize that the concept of prime numbers applies only to whole numbers. A "**composite**" number is a number with more than two factors and includes all non-prime numbers other than one and zero (e.g., 9 has factors of 1, 3, 9). It is important for students to realize that 0 and 1 are not classified as a prime or composite numbers**.** The number "one" has only one factor (itself). Zero is not prime because it has an infinite number of divisors and it is not composite because it cannot be written as a product of two factors that does not include 0.

Although students should have strategies for determining whether or not a number is prime, it is not essential for them to be able to quickly recognize prime numbers. However, students should be able to readily identify even numbers (other than 2) as non-primes (composites) as they will have a minimum of three factors: 1, 2 and the number itself.

Students should be encouraged to accurately use language such as multiple, factor, prime and composite. As well, encourage students to explore numbers and become familiar with their composition.

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:
- Unit 2, Lesson 3, pp. 55-58
- Unit 2, Lesson 4, pp. 59-62
- Unit 2, Lesson 5, pp. 63-66
- Unit 2, Game, p. 67
- Unit 2, Lesson 6, pp. 68, 69

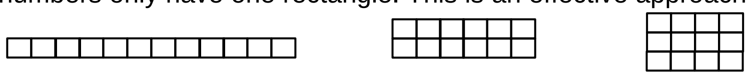
SCO: **6.N2 Demonstrate an understanding of factors and multiples by:**
- **determining multiples and factors of numbers less than 100**
- **identifying prime and composite numbers**
- **solving problems involving multiples.**

[PS, R, V]

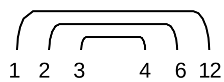
Instructional Strategies

Consider the following strategies when planning lessons:

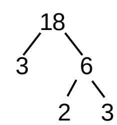
- Have students determine the factors of a number by arranging square tiles into as many different arrays (rectangles) as possible. Record the length and width of each rectangle. For example, if 12 tiles were used, the rectangles would be 1 by 12, 2 by 6, and 3 by 4. These are the factor pairs for 12. Have students record their rectangles/ factor partners on grid paper. Students should discover that some numbers only have one rectangle. This is an effective approach to introducing prime numbers.



- Have students investigate other numbers to find their factor pairs. Students may use organized lists to determine factors (i.e., begin with 1 and the number itself, then 2 or the next possible factor and its factor partner, etc.)



- Have students factor odd composite numbers (e.g., 33, 39). Students sometimes mistake these for prime as they do not readily see how they are factored.
- Have students use various Cuisenaire® rods or connected base ten unit cubes on a meter stick to find multiples of a number. Have students list the multiples found and relate the patterns to the times table.
- Have students explore other strategies such as factor trees to determine prime and composite numbers.



Suggested Activities

- Explore the sieve of Eratosthenes to identify the prime numbers to 100. On a hundreds chart, have the students begin by circling the first prime number, 2, and then cross out all the multiples of 2 (composite numbers). Circle the next prime number, 3, and cross out all of its multiples. Students then proceed to the next number that has not been crossed off and repeat the procedure. At the end of the process the circled numerals will be the prime numbers up to 100. Discuss any patterns they notice.
- Have students express even numbers greater than 2 in terms of sums of prime numbers. (Sample answers may include 4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, ..., 48 = 43 + 5, 50 = 47 + 3, ...). Explore this idea further by asking if every even number greater than 2 can be written as the sum of 2 primes (Goldbach's Conjecture).
- Ask students to name numbers with a given amount of factors (e.g., numbers with 6 factors: 12, 18, 20, etc.).
- Have students use the constant function on their calculators to explore multiples of a number. They may also use calculators to systematically test for factors of a number: ÷ 1, ÷ 2, ÷ 3, ÷ 4, etc.

SCO: **6.N2 Demonstrate an understanding of factors and multiples by:**
- **determining multiples and factors of numbers less than 100**
- **identifying prime and composite numbers**
- **solving problems involving multiples.**

[PS, R, V]

Assessment Strategies

- Ask students to express 36 as the product of two factors in as many ways as possible.
- Have small groups of students find the number less than 50 (or 100) that has the most factors. Ensure students can explain their process and justify their answer.
- Have students show all the factors of 48 by drawing or colouring arrays on square grid paper.
- Ask students if it is possible to list all of the multiples of 12? Explain their reasoning.
- Have students list all of the factors of 8 and some of the multiples of 8.
- Ask students to explain, without dividing, that 2 cannot be a factor of 47.
- Ask students to identify a number with 5 factors.
- Ask students to find 3 pairs of prime numbers that differ by two (e.g., 5 and 7).
- Ask students: Why is it easy to know that certain large numbers (e.g., 4 283 495) are not prime, even without factoring them?
- Tell the students that the numbers 2 and 3 are consecutive numbers, both of which are prime numbers. Ask: Why can there be no other examples of consecutive prime numbers?
- Have students use a computer or calculator to help them determine the prime numbers up to 100. Ask them to prepare a report describing as many features of their list as they can.
- Have students draw diagrams (such as rectangles or factor rainbows) to show why a given number is or is not prime (e.g., 10, 17, 27).

SCO: **6.N3 Relate improper fractions to mixed numbers.**
[CN, ME, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.N7 Demonstrate an understanding of fractions by using concrete and pictorial representations to: create sets of equivalent fractions; compare fractions with like and unlike denominators.	**6.N3** Relate improper fractions to mixed numbers.	**7.N6** Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using: • benchmarks; • place value; • equivalent fractions and/or decimals.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. demonstrate using models that a given improper fraction represents a number greater than 1;
B. express improper fractions as mixed numbers;
C. express mixed numbers as improper fractions; and
D. place a given set of fractions, including mixed numbers and improper fractions, on a number line and explain strategies used to determine position.

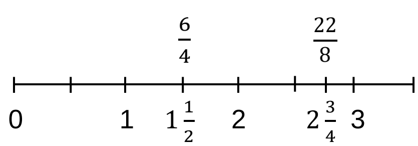
SCO: **6.N3 Relate improper fractions to mixed numbers.**
[CN, ME, R, V]

Elaboration

In Grade 6, students extend their understanding of fractions to learn that an **improper fraction** represents a fraction greater than or equal to one. Through the use of models, students should discover that fractions with the numerator greater than their denominator are greater than one (e.g., $\frac{5}{3},\frac{6}{2},\frac{7}{6}$). It is important for students to understand that an improper fraction can also be expressed as a **mixed number** which is a whole number and a **proper fraction** (e.g., $1\frac{1}{4}$).

Students should fluently move between the mixed number and improper fraction formats of a number. Rather than only applying a rule to move from one format to the other, students should be encouraged to focus on the meaning. For example, since $\frac{14}{3}$ is 14 thirds and it takes 3 thirds to make 1 whole, 12 thirds would equal 4 wholes, so $\frac{14}{3}$ represents 4 wholes and another 2 thirds of another whole or $4\frac{2}{3}$. Often it is easier for students to grasp the magnitude of mixed numbers than improper fractions. For example, a student may know that $4\frac{1}{3}$ is a bit more than 4, may not have a good sense of the size of $\frac{13}{3}$.

Students should be able to place mixed numbers and improper fractions on a number line easily when they have **benchmarks** to use such as: closer to zero, close to one half, closer to one, etc. Having these benchmarks helps students visualize the placement and order of these fractions. The concept of equivalent fractions that students learned in grade 5 will also be helpful in developing additional benchmarks.



It is important that students have an opportunity to explore that fractions are connected to multiplication and division through a problem-solving context and the use of a variety of models. Students should discover that dividing the numerator by the denominator is a procedure that can be used to change an improper fraction to a mixed number. It would be inappropriate just to tell students to divide the denominator into the numerator to change an improper fraction to a mixed number before they develop the conceptual understanding for this.

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:

- Unit 5, Lesson 1, pp. 162-165
- Unit 5, Lesson 2, pp. 166-169
- Unit 5, Game, p. 170
- Unit 5, Lesson 3, pp. 171-175
- Unit 5, Lesson 6, pp. 184, 185
- Unit 5, Unit Problem, pp. 196, 197

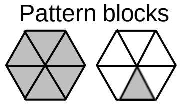
SCO: **6.N3 Relate improper fractions to mixed numbers.**
[CN, ME, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

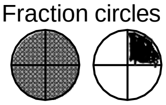
- Have students explore improper fractions and mixed numbers in a variety of ways and use different models. Some examples are:

Pattern blocks



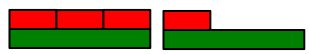
$1\frac{1}{6} = \frac{7}{6}$

Fraction circles



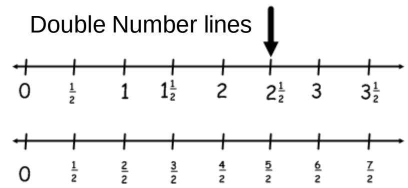
$1\frac{1}{4} = \frac{5}{4}$

Cuisenaire® rods

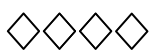


$1\frac{1}{3} = \frac{4}{3}$

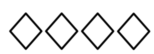
Double Number lines



- Have students use pattern blocks and have students build and count fractional parts and continue beyond a whole: $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}$, etc. Ask them to show another way to represent the improper fractions (e.g., $\frac{5}{3} = 1\frac{2}{3}$). Gradually transition to doing this activity without the pattern blocks (or other models).
- Provide students with frequent opportunities to use number lines (including double number lines) to explore the placement of mixed numbers and improper fractions. Ensure students are able to explain their strategy focusing on the use of benchmarks.
- Have students model $\frac{9}{4}$ and tell how many groups of 4 are in 9. For example;



1st group of 4

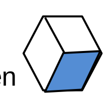


2nd group of 4

$\frac{1}{4}$ of a group of 4

Suggested Activities

- Ask students to model mixed numbers and improper fractions in various ways (e.g., $1\frac{3}{4} = \frac{7}{4}$).



- Have students determine what fraction the blue rhombus represents if the hexagon is the whole. When this task is complete have the students explain using the pattern blocks what another name for $\frac{14}{3}$ is?
- Have students solve problems such as: Jamir has 15 quarters in his pocket. How many whole dollars does he have?
- Create a set of equivalent mixed number and improper fraction cards and distribute a card to each student. Students need to find their equivalent partner. Then have students line up by pairs in ascending order (a temporary number line on the floor might be helpful for students). This activity should be done after students have had opportunity to develop their understanding with models.

SCO: **6.N3 Relate improper fractions to mixed numbers.**
[CN, ME, R, V]

Assessment Strategies

- Ask students: If 14 people at a party each want $\frac{1}{3}$ of a pizza, how many pizzas would be needed?
- Ask students to use coloured squares to show why $3\frac{1}{3} = \frac{10}{3}$. Observe whether or not they make wholes of 3 (or 6 or 9...) squares.
- Provide students with several mixed numbers and improper fractions that are equivalent (e.g., $2\frac{3}{4} = \frac{11}{4}$). Ask them to show if the numbers are equal and to explain their thinking concretely, pictorially and symbolically.
- Provide students with several mixed numbers and improper fractions. Have students place the numbers on an open number line to demonstrate their relative magnitude.
- Write and model a mixed number, with the same denominator, that is greater than $\frac{3}{3}$, but less than $\frac{6}{3}$.

SCO: **6.N4 Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.N9 Relate decimals to fractions and fractions to decimals (to thousandths).	**6.N4** Demonstrate an understanding of percent, (limited to whole numbers) concretely, pictorially and symbolically.	**7.N3** Solve problems involving percents from 1% to 100%.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. explain that “percent” means “out of 100;”

B. explain that percent is a ratio out of 100;

C. use concrete materials and pictorial representations to illustrate a given percent;

D. record the percent displayed in a given concrete or pictorial representation;

E. express a given percent as a fraction and a decimal;

F. identify and describe percents from real-life contexts, and record them symbolically; and

G. solve a given problem involving percents.

SCO: **6.N4 Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

Elaboration

Percent is a part-to-whole ratio that compares a number to 100. “Percent” means “out of 100” or “per 100”. Students should understand that percent on its own does not represent a specific quantity. For example, 90% might represent 9 out of 10, 18 out of 20, 45 out of 50, and 90 out of 100.

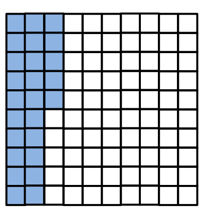
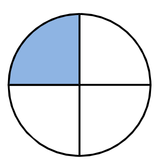
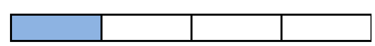
Percent can always be written as a decimal or vice versa. For example, 26% is the same as 0.26, and both mean 26 hundredths or $\frac{26}{100}$.

Students should recognize:
- situations in which percent is commonly used;
- diagrams, showing parts of a set, whole, or measure that represent various percentages (e.g., 2%, 35%);
- the relationship between the percent and corresponding decimals and ratios (e.g., 48%, 0.48, 48:100);
- the percent equivalents for common fractions and ratios such as $\frac{1}{4}$ = 25%, $\frac{1}{2}$ = 50%, and $\frac{3}{4}$ = 75%.

Students do **not** need to compute with percentages or work with percentages greater than 100 in grade 6.

Number sense for percent should be developed through the use of these basic **benchmarks**:
- 100% is all;
- 50% is half;
- 25% is a quarter; 75% is three quarters;
- 33% is a little less than a third; 67% is a little more than two thirds.

It is important for students to use a variety of representations of percent to help deepen their understanding. For example, 25% can be represented as shown below.

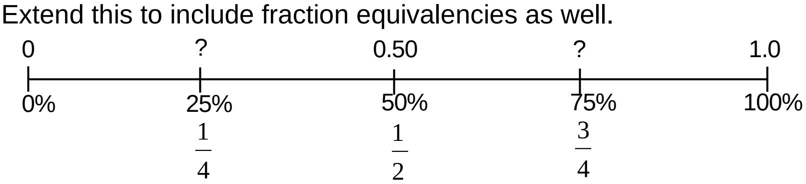
This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:
- Unit 5, Lesson 7, pp. 96-99
- Unit 5, Lesson 8, pp. 190-193
- Unit 5, Unit Problem, pp. 196, 197

SCO: **6.N4 Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

Instructional Strategies

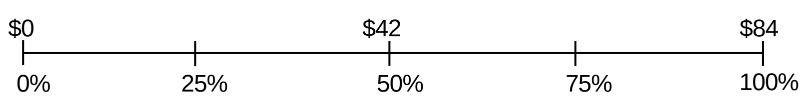
Consider the following strategies when planning lessons:

- Provide students with many opportunities to work with partially shaded hundreds grids, determining the decimal, fraction, ratio, and percent that is shaded.
- Make charts, including symbolic representations, for fractions, decimals, and percents that are equal.
- Use virtual manipulatives available on the Internet and Interactive whiteboard software.
- Have students predict percentages, give their prediction strategies, and then check their predictions. For example, ask them to estimate the percentage of:
  - each colour of Bingo chips, if a total of 100 blue, red, and green chips are shown on an overhead for 10 seconds;
  - a hundredths grid that is shaded in to make a picture;
  - red counters when 50 two coloured counters are shaken and spilled.
- Use a double number line as a useful tool to model and solve simple percent equivalencies and problems. Extend this to include fraction equivalencies as well.



Suggested Activities

- Ask students to draw a design on a hundred grid and describe the percent that is shaded.
- Have students create a pencil crayon quilt made of patches of various colours. They can describe the approximate or exact percentages of each colour within the patch and then estimate the percent of the total quilt that is each colour.
- Tell students that Jane is covering her floor with tiles. The whole floor will take $84 worth of tiles. How much will she have spent on tiles when 25% of her floor is covered? Use a number line to help model.

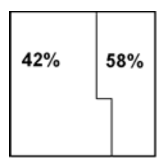


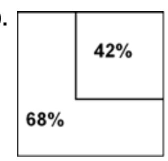
- Have students collect examples of situations from newspapers, flyers, or magazines in which percent is used and have them make a collage for a class display.
- Have students estimate the percent of time students spend each day doing certain activities (e.g., attending school, physical activity, eating, sleeping, etc.).
- Have students estimate and then determine the percentage of pages in a magazine that have advertisements on them.

SCO: **6.N4 Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

Assessment Strategies

- Ask students:
  a. Which is the least? The most? Explain your answer.
  $\frac{1}{20}$, 20%, 0.02
  b. Which one doesn't belong? Explain your choice:
  $\frac{3}{4}$, 0.75, 0.34, 75%
- Ask students what percent of a meter stick is 37 cm?
- Have students examine a set of object and describe different ratio and percent equivalents.
- Have students name percents that indicate:
  - almost all of something
  - very little of something
  - a little less than half of something
- Ask students what is incorrect about each of the following diagrams:
  Have students justify their answers.

a.


b.


SCO: **6.N5 Demonstrate an understanding of integers, concretely, pictorially and symbolically.** [C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Five	Grade Six	Grade Seven
	6.N5 Demonstrate an understanding of integers, concretely, pictorially and symbolically.	**7.N4** Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. extend a given number line by adding numbers less than zero and explain the pattern on each side of zero;

B. place given integers on a number line and explain how integers are ordered;

C. describe contexts in which integers are used, e.g., on a thermometer;

D. compare two integers, represent their relationship using the symbols <, > and =, and verify using a number line; and

E. order given integers in ascending or descending order.

SCO: **6.N5 Demonstrate an understanding of integers, concretely, pictorially and symbolically.** [C, CN, R, V]

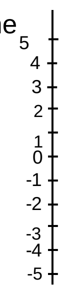
Elaboration

Negative numbers have been part of the day-to-day life of students through their experiences such as temperatures below zero. Students will be introduced to the set of integers which includes positive and negative whole numbers and zero.

The big ideas of integers that students should understand in grade 6 are:
- each negative integer is the mirror image of a positive integer with respect to the 0 mark therefore the same distance from zero;
- 0 is neither positive nor negative;
- negative integers are all less than any positive integer;
- a positive integer closer to zero is always less than a positive integer farther away from zero (e.g., +3 < +7);
- a negative integer closer to zero is always greater than a negative integer farther away from zero. (e.g., -3 > -7).

Students should be encouraged to read -5 as “negative 5” rather than “minus 5,” to minimize confusion with the operation of subtraction. It is also important for students to recognize that positive integers do not always show the “+” symbol. If no symbol is shown, the integer is positive.

Students will have previously encountered negative integers in several of the above situations, but one of the most common contexts is a thermometer. To build on this informal understanding, it is beneficial to start with a vertical number line which resembles a thermometer.



Other useful contexts for considering negative integers are:
- temperatures;
- elevators which go both above and below ground (floors can have positive and negative labels);
- golf scores above and below par;
- money situations involving debits and credits;
- distance above and below sea level.

In prior grades, students will have compared numbers using the vocabulary of “**greater than**” and “**less than**”. In grade 6, students will be expected to represent these comparisons using the **>** and **<** symbols.

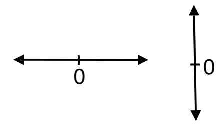
Addition and subtraction situations involving integers should only be explored informally as it is a grade 7 outcome.

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:
- Unit 2, Lesson 8, pp. 74-77
- Unit 2, Lesson 9, pp. 78-81
- Unit 2, Unit Problem, pp. 84, 85

SCO: **6.N5 Demonstrate an understanding of integers, concretely, pictorially and symbolically.**
[C, CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:



- Provide students with an open number line to explore the placement of integers.
- Explore examples of situations where negative integers are used from various media.
- Have students divide a sheet of paper into 3 parts with the headings of Negative, Positive, and Zero. As situations arise throughout this outcome, have students record the situation under the headings which best describes it. For example, rise in temperature (positive), spending money (negative), freezing point (zero).
- Give each student a card with an integer number on it (ensure that the set of cards includes pairs of integers, such as +7, -7, and a card with zero). Have the person with the “zero” card stand at the front of the classroom in the middle. Have the rest of the students create a “human number line” placing them in order according to the card they were given.
- Use a thermometer (vertical number line) to compare integers and record the comparison symbolically (-8 < 5; 6 > -7; 4 < 9; -3 > -4).

Suggested Activities

- Have 10 students volunteer, to come to the front of the class. They are given an integer unknown to them, on a sticky note, stuck on their backs. The volunteers without talking must rearrange themselves in ascending by moving each other.
- Have students place a variety of integers at the appropriate places on a number line.
- Have students play the card game “integer war”, using the red cards for negative integers and the black cards as positive integers. Each student flips a card, the student holding the card with the highest value, wins both cards.
- Have students choose 10 cities and research the temperature for a specific date, enter the data into a table from warmest to coldest temperatures. Students may use a vertical number line to facilitate this task.
- Have students write an integer for each of the following situations:
  a. A person walks up 8 flights of stairs.
  b. An elevator goes down 7 floors.
  c. The temperature falls by 7 degrees.
  d. Josh deposits $110 dollars in the bank.
  e. The peak of the mountain is 1123 m above sea level.
- Have students investigate **opposite integers** by plotting points such as +5 and -5 on a number line. What do you notice about them? Why do you think number pairs such as -5 and +5 are called opposites?

SCO: **6.N5 Demonstrate an understanding of integers, concretely, pictorially and symbolically.**
[C, CN, R, V]

Assessment Strategies

- Ask students: How many negative integers are greater than -7?
- Tell students that a number is 12 jumps away from its opposite on a number line. Ask: What is the number?
- Have students explain why -4 and +4 are closer to each other than -5 and +5.
- Ask students to design a simple game for which positive and negative points might be awarded. Have the students play and keep track of their total scores.
- Ask students why is an integer never 11 away from its opposite on a number line?
- Have students flip over two playing cards (red cards could represent negative integers and black cards could represent positive integers). Record the comparison symbolically with numbers and the symbols > and <.
- Ask students to explain why it is true that:
  a. a negative number further from zero is less than a negative number that is close to zero;
  b. a negative number is always less than a positive number;
  c. a positive number is always greater than a negative number;
  d. integer opposites cancel each other out when they are combined.

SCO: **6.N6 Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).**
[C, CN, ME, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.N5 Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems. **5.N6** Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems. **5.N11** Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).	**6.N6** Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).	**7.N2** Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. place the decimal point in a product using front-end estimation, e.g., for 15.205 m × 4, think 15 m × 4, so the product is greater than 60 m;
B. place the decimal point in a quotient using front-end estimation, e.g., for $25.83 ÷ 4, think $24 ÷ 4, so the quotient is greater than $6;
C. correct errors of decimal point placement in a given product or quotient without using paper and pencil;
D. predict products and quotients of decimals using estimation strategies; and
E. solve a given problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.

SCO: **6.N6 Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).**
[C, CN, ME, PS, R, V]

Elaboration

Students will have had experience multiplying and dividing whole numbers in previous grades. The emphasis will continue to be on the understanding of these two operations rather than the mastery of one traditional algorithm. As students extend their learning to multiplying and dividing with decimals the use of estimation is essential to help students ensure the reasonableness of their answer. "When estimating, thinking focuses on the meaning of the numbers and the operations, and not on counting decimal places" (Van de Walle & Lovin, vol. 3, 2006, p. 125).

When considering multiplication by a decimal, students should recognize that, for example, 0.8 of something will be almost that amount, but not quite, and 2.4 multiplied by an amount will be double the amount with almost another half of it added on. It is important for students to realize estimation is a useful skill in their lives and regular emphasis on real-life contexts should be provided. On-going practice in computational estimation is a key to developing understanding of number and number operations and increasing mental process skills. Although rounding has often been the only estimation strategy taught, there are others (many of which provide a more accurate answer) that should be part of a student's repertoire such as front-end estimation:

- Multiplication: 6 × 23.4 might be considered to be 6 × 20 (120) plus 6 × 3 (18) plus a little more for an estimate of 140, or 6 × 25 = 150.
- Division: Pencil and paper division involves front-end estimation. For, 424.53 ÷ 8 (or), $8\overline{)424.53}$ students should be able to estimate that 50 × 8 is 400, so the quotient must be a bit more than 50.

Students should be able to place missing decimals in products and quotients using estimation skills and not rely on a rule for "counting" the number of digits without understanding.

A connection should be made between multiplication and division. Multiplication can be used to estimate **quotients**. For example, 74.3 divided by 8. Have a student say the multiples of 8 that are closest to 74.3. Write out 8 × 9 = 72 and 8 × 10 = 80. Students should explain how they know the quotient is between 9 and 10. Ensure proper vocabulary when reading all numbers. This will assist students in making the connection between facts (e.g., 4 × 6 is similar to 4 × 0.6; 4 groups of 6 tenths = 24 tenths or 2.4).

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to the Grade 6 Mathematics page at learn.edu.pe.ca for the Mental Math Guide.

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:

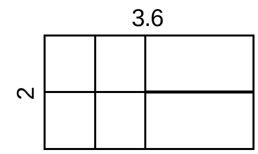
- Unit 3, Lesson 2, pp. 92-94
- Unit 3, Lesson 3, pp. 95-98
- Unit 3, Lesson 4, pp. 99-102
- Unit 3, Lesson 5, pp. 103-107
- Unit 3, Lesson 6, pp. 108-111
- Unit 3, Lesson 7, pp. 112-114
- Unit 3, Game, p. 115
- Unit 3, Lesson 8, pp. 116, 117
- Unit 3, Unit Problem, pp. 120, 121

SCO: **6.N6 Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).**
[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure students use proper vocabulary related to multiplication (factors, product) and division (divisor, dividend, quotient) that they have learned in previous grades.
- Have students look for benchmark decimals that are easy to multiply and divide. For example, Ask students why someone might estimate 516 × 0.48 by taking half of 500.
- Provide opportunities for students to create and solve missing factor and missing divisor/dividend problems, involving decimals, to support the connection between multiplication and division.
- Use the "area model" both concretely with base ten blocks and pictorially to represent multiplication and division before moving to the symbolic. For example, 2 × 3.6 could be modelled as:



Suggested Activities

- Provide students with a number sentence that has decimals missing or misplaced in either the answer or the question. For example, 2.34 × 6 = 1404 a decimal is missing in the product. Have students determine where the decimal should be using estimation strategies such as "front-end".
- Have students estimate each of the following and tell which of their estimates is closer and how they know: 3 videos games at $24.30/game OR 5 teen magazines at $8.89/magazine;
  9 glasses of fruit smoothies at $2.59/glass OR 4 veggie pitas at $4.69/pita.
- Tell the students that it takes about 9 g of cookie dough to make one cookie. Renee checks the label on the package and finds she has 145.6 g of dough. About how many cookies can she make?
- Have students measure side lengths of objects in the classroom to the nearest tenth of a centimetre or hundredth of a metre and then estimate the area of those objects (e.g., side lengths of their desks, their textbooks or the top of tables).
- Have students solve problems which involve dividing the price for a pizza. For example, 4 people sharing a pizza for $14.56. Change the amount of people and the price of the pizza for more problems.
- Tell the students that the cashier told Samantha that her total for 3 kg of grapes at $3.39/kg was $11.97. How did Samantha use estimation to know that the cashier had made a mistake?
- Provide real-world problems involving multiplication and division of decimals where the multiplier/divisor are 1-digit whole numbers. For example, Jean works at Pizza Pie for $8.75/hour. Saturday he worked 8 hours. What were his earnings? Sunday he made $93.25, and was paid $9.00 per hour. How many hours did he work?
- Have students figure out how much they need to pay, if they went to the restaurant with three friends and the bill came to $26.88. Students should assume that each person pays their equal share.

SCO: **6.N6 Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).**
[C, CN, ME, PS, R, V]

Assessment Strategies

- Tell students that you have multiplied a decimal by a whole number and the estimated product is 5.5. What might the two numbers be?
- Provide students with a supermarket checkout slip and tell them that it represents a family's weekly groceries. Have students estimate the total amount spent per day or per month by that family.
- Ask students for an estimate of the total cost of 8 pens at $0.79 each. Ask what estimating strategy he/she used and if there is another way to easily estimate the answer.
- Have students estimate the mass of each egg in kilograms, if they know that the total mass of a half dozen eggs is 0.226 kg.
- Ask students to identify which of the following is the best estimate for 13.7 × 9 and explain why.
  13.0 × 9   4.0 × 9   15.0 × 9   14.0 × 10   10.0 × 9

SCO: **6.N7 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).**
[CN, ME, PS, T]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Five	Grade Six	Grade Seven
	6.N7 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).	**7.N2** Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals to solve problems.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. demonstrate and explain with examples why there is a need to have a standardized order of operations; and
B. apply the order of operations to solve multi-step problems with or without technology, e.g., computer, calculator.

SCO: **6.N7 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).**
[CN, ME, PS, T]

Elaboration

Students should realize that the convention for **order of operations** is necessary in order to maintain consistency of results in calculations. It is important to provide students with situations in which they can recognize the need for the order of operations.

The purpose of the order of operations is to ensure that the same answer is reached regardless of who performs the calculations. When more than one operation appears in an expression or equation, the operations must be performed in the following order:
- operations in brackets first;
- divide or multiply from left to right whichever operation comes first;
- add or subtract from left to right whichever operation comes first.

The acronym, “BEDMAS”**,** is a common memory device to recall the order of operations. It is important to stress that even though the “D” appears before the “M” and the “A” before the “S”, these pairs of operations are done in the order that they appear (multiplication or division, then addition or subtraction). The “E” represents exponents, however, this is not a concept that is expected of Grade 6 students. It may be helpful to have students develop their own method of recalling the order of operations.

Students should be taught that brackets may also be referred to as **parentheses** and can have different shapes: **( )**, [ ], { }. The specialized meanings of each of these types will be explored in more depth in later grades and the focus in Grade 6 order of operations should be on using first type listed: ( ). Some calculators have brackets that can be entered during calculations and the use of this function could be used by students.

When solving multi-step problems, it is important for students to recognize when it is appropriate to use technology. Students should be encouraged to use mental math and computational skills as much as possible. Students should be able to solve many multi-step problems mentally, such as $50 \times (12 \div 4)$.

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:
- Unit 2, Lesson 7, pp. 70-73

SCO: **6.N7 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).**
[CN, ME, PS, T]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students work in groups to answer the following: 8 – 2 x 4 + 10 ÷ 2, then share their answers. Discuss why some found different answers and the need for rules so we all get the same answer. This could be extended by asking students where brackets could be placed to get the largest or smallest possible answer.
- Apply the rules for order of operations by modeling a variety of problem solutions. Students can check to determine whether their calculator follows the rules for order of operations. Depending on their type calculators may yield different results.
- Have students become human models of the numbers in a problem and others students could become brackets as necessary to get an answer that was previously chosen. Students need to move so that the answer will be produced.
- Ask students to write a number sentence for the following: the total cost for a family with two parents and three children for theatre tickets if children's tickets cost $9 and adult tickets cost $12. When students write a number sentence such as, 3 × $9 + 2 × $12, ask if this solution makes sense:
  3 × $9 = $27 + 2 = $29 × $12 = $348.

Suggested Activities

- Have students write number sentences for the following problems and solve them using the order of operations. Consider solving the number sentences for a) and b) by ignoring the order of operations. Would the solution make sense in terms of the problem? Discuss.
  a. Ms. Janes bought the following for her project: 5 sheets of pressboard at $9 a sheet, 20 planks at $3 each, and 2 litres of paint at $10. What was the total cost?
  b. Three times the sum of $35 and $49 represents the total amount of Jim's sales on April 29. When his expenses, which total $75, were subtracted, what was his profit?
- Tell students that Billy had to answer the following skill-testing questions to win the contest prize. What are the winning answers? a. 234 × 3 – 512 ÷ (2 × 4) b. 18 + 8 × 7 – 118 ÷ 4
  Billy was told that the correct answer for "b" is 16, but Billy disagreed. What did the contest organizers do in solving the question which caused them to get 16 for the answer? Explain why you think they made that error.
- Ask students to explain why it is necessary to know the order of operations to compute 4 × 7- 3 × 6. Ask them to compare the solution of the previous problem with the solution of 4 × (7 - 3) × 6. Ask whether the solutions are the same or different and why.
- Provide students with a set of numbers and a target solution. Have students explore and discover where they can place operations symbols and brackets to achieve the solution. For example:
  3, 6, 3, 4. Solution = 11 3,6,3,4 Solution = 108 3,6,3,4 Solution = 6

Possible answers: 3 + (6 ÷3) × 4	(3 + 6) × (3 × 4)	(3 × 6) – (3 × 4)

SCO: **6.N7 Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).**
[CN, ME, PS, T]

Assessment Strategies

- Tell students that as a result of some faulty keys, the operation signs in these problems did not print. Use the information which is supplied to help determine which operations were used.
  a. (7 □ 2) □ 12 = 2 b. (12 □ 4) □ 4 = 7
- Tell students that because the shift key on the keyboard did not work, none of the brackets appeared in the following problems. If the student has the right answers to both problems, identify where the brackets must have been.
  a. 4 + 6 × 8 - 3 = 77 b. 26 - 4 × 4 – 2 = 18
- Have students use their calculator to answer the following question: Chris found the attendance reports for hockey games at the stadium to be 2787, 2683, 3319, 4009, 2993, 3419, 4108, 3539, and 4602. If tickets were sold for $12 each, and expenses amounted to $258 712, what was the profit for the stadium? Have the students write out the equation to demonstrate their understanding of order of operations.
- Have students create their own multi-operation expression, including brackets, and show its solution.
- Have students solve an order of operations expression and then describe what could have gone wrong if the order of operations steps were not followed (e.g., what might be an incorrect solution).

PATTERNS AND RELATIONS

SPECIFIC CURRICULUM OUTCOMES

Patterns

6.PR1 – Demonstrate an understanding of the relationship within tables of values to solve problems.

6.PR2 – Represent and describe patterns and relationships using graphs and tables.

Variables and Equations

6.PR3 – Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.

SCO: **6.PR1 Demonstrate an understanding of the relationships within tables of values to solve problems.**
[C, CN, PS, R]
SCO: **6.PR2 Represent and describe patterns and relationships using graphs and tables**.
[C, CN, ME, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.PR1 Determine the pattern rule to make predictions about subsequent terms (elements).	**6.PR1** Demonstrate an understanding of the relationships within tables of values to solve problems. **6.PR2** Represent and describe patterns and relationships using graphs and tables.	**7.PR1** Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.

Achievement Indicators

Students who have achieved this outcome(s) should be able to:
6.PR1

A. generate values in one column of a table of values, given values in the other column and a pattern rule;
B. state, using mathematical language, the relationship in a given table of values;
C. create a concrete or pictorial representation of the relationship shown in a table of values;
D. predict the value of an unknown term using the relationship in a table of values and verify the prediction;
E. formulate a rule to describe the relationship between two columns of numbers in a table of values;
F. identify missing elements in a given table of values;
G. identify errors in a given table of values;
H. describe the pattern within each column of a given table of values; and
I. create a table of values to record and reveal a pattern to solve a given problem.

6.PR2

A. translate a pattern to a table of values and graph the table of values (limit to linear graphs with discrete elements);
B. create a table of values from a given pattern or a given graph; and
C. describe, using everyday language, orally or in writing, the relationship shown on a graph.

SCO: **6.PR1 Demonstrate an understanding of the relationships within tables of values to solve problems.**
[C, CN, PS, R]
SCO: **6.PR2 Represent and describe patterns and relationships using graphs and tables**.
[C, CN, ME, PS, R, V]

Elaboration

Mathematics is often referred to as the study of patterns, as they permeate every mathematical concept and are found in everyday contexts. The various representations of patterns including **physical models**, **table of values**, **algebraic expressions**, and **graphs**, provide valuable tools in making generalizations of mathematical relationships.

Patterns include **repeating** patterns and **growing** patterns. An example of a repeating pattern is 1, 2, 2, 1, 2, 2, 1, 2...). Growing patterns include **arithmetic** (adding or subtracting the same number each time) and **geometric** (multiplying or dividing the same number each time) situations. Patterns using concrete and pictorial representations can be written using numbers, where numbers represent the quantity in each step of the pattern.

A **table of values** shows the relationship between pairs of numbers. Students should use tables to organize and graph the information that a pattern provides. When using tables, it is important for students to realize that they are looking for the relationship between two variables (**term number and term**). The **relationship** tells what you do to the previous term to get the next term. The **pattern rule** is what you do to the term number to get the term value. For example, the number pattern 1, 3, 5, 7, 9,…has the relationship where each number increases by two. The rule for this pattern is $2n$ -1.

Term number (n)	1	2	3	4	5
Term (2n-1)	1	3	5	7	9

Input	Output
1	1
2	3
3	5
4	7
5	9

The analysis of graphs should include creating "stories" or real-world situations that describe the relationship depicted. Similarly, when constructing graphs, a story that matches the changes in related quantities should be included. When students are describing a relationship in a graph they should use language like: as this increases that decreases; as one quantity drops, the other also drops, etc.

Students should be able to create a table of values for a given linear relationship and be able to match graphs and sets of linear relationships. This concept is connected to outcomes **6.**SP1 and **6.**SP3.

6.PR1: This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:

- Unit 1, Lesson 1, pp. 6-10
- Unit 1, Lesson 2, pp. 11-15
- Unit 1, Lesson 3, pp. 16, 17
- Unit 1, Game, p. 18
- Unit 1, Lesson 4, pp. 19-23
- Unit 1, Unit Problem, pp. 42, 43

6.PR2: This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:

- Unit 1, Lesson 4, pp. 19-23
- Unit 1, Lesson 6, pp. 29-32
- Unit 1, Unit Problem, pp. 42, 43

SCO: **6.PR1 Demonstrate an understanding of the relationships within tables of values to solve problems.**
[C, CN, PS, R]
SCO: **6.PR2 Represent and describe patterns and relationships using graphs and tables**.
[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Develop a table with incorrect values and a correct pattern rule. Have students become “Data Detectives”, finding and correcting the errors.
- Have students create the following pattern with counters, develop a table of values to display the information, write the relationship, and then graph it. Have students predict the value of unknown terms.

- Provide students with graphs to analyze and have them create corresponding tables of values. Have them describe the relationship shown in the graph orally or in writing.

Suggested Activities

- Tell students about a family vacation situation. The family drove for 5 hours the first day and covered 450 km. The second day the family went 8 hours and 720 km. The last day they arrived in Las Vegas after 6 hours (540 km). Have students create a table of values for this data, describe the pattern, and make a graph.
- Have students fill in the blanks of the tables below and then state the relationship and write the rule.

Numerator	?	2	3	?	5
Denominator	?	8	12	?	?

Side Length (cm)	1	2	3	4	5	6	
Perimeter (cm)	6	12	18		30		48

Input	Output
1	2
2	4
3	
4	8
	10

- Ask students to create a concrete and pictorial display of a table of values showing the balance in a bank account or the height of a plant as it grows. Have students graph the information.
- Describe a real-world situation to the students that depicts a pattern. For example, a taxi ride costs $2.50 to start and then $0.40 for each kilometre. How much does to cost to travel 1 km? 2 km? 3 km?. Have students record the pattern, create a table of values, and graph the relationship. Have them determine the total cost of a 15 km trip.
- Have students identify the relationship, rule, and state the value for the 3rd and 12th terms for a given table.

SCO: **6.PR1 Demonstrate an understanding of the relationships within tables of values to solve problems.**
[C, CN, PS, R]
SCO: **6.PR2 Represent and describe patterns and relationships using graphs and tables**.
[C, CN, ME, PS, R, V]

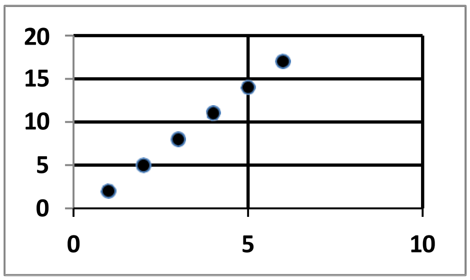
Assessment Strategies

- Ask students to put the numbers 2, 4, 4, 5, 12, 20 and 40 in the correct spots in the tables of equivalent fractions shown below.

Numerator	1		3	
Denominator	4	8		16

Numerator	2		8	16
Denominator		10		

- Have students create the table of values from a graph, such as the one below, and describe the relationship in words.



- Have students refer to the following table to answer these questions:
  a. Create a rule showing how the number of hours of lessons can help determine the cost for one lesson if you pay at the start to rent skis. Explain your thinking.
  b. Use this rule to predict the total cost for 10 hours of lessons.
  c. Why do 10 hours not cost twice as much as 5 hours?
  d. Create a graph to show the values in the table.

Cost of Ski Lessons and Rental	
Number of hours (h)	Cost ($)
1	80
2	110
3	140
4	170
5	200

- Provide a visual pattern such as the one below. Have students create and graph its table of values and describe the relationship.

SCO: **6.PR3 Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.PR2 Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.	**6.PR3** Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.	**7.PR2** Demonstrate an understanding of preservation of equality by: **7.PR4** Evaluate an expression given the value of the variable(s).

Achievement Indicators

Students who have achieved this outcome should be able to:

A. model the preservation of equality for addition using concrete materials, such as a balance or using pictorial representations and orally explain the process;

B. model the preservation of equality for subtraction using concrete materials, such as a balance or using pictorial representations and orally explain the process;

C. model the preservation of equality for multiplication using concrete materials, such as a balance or using pictorial representations and orally explain the process;

D. model the preservation of equality for division using concrete materials, such as a balance or using pictorial representations and orally explain the process; and

E. write equivalent forms of a given equation by applying the preservation of equality and verify using concrete materials, e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$.

SCO: **6.PR3 Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

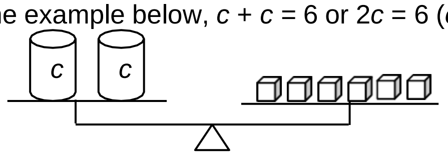
Elaboration

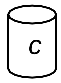
Students have had experience exploring the concept of equality since grade 2 and solving equations in a basic form since grade three. A misconception for some students may be that the equal sign indicates an answer. They will need further practice and reinforcement in grade 6 to view the equal sign as a symbol of **equivalence** and balance, and represents a **relationship**, not an operation.

Through the use of balance scales and concrete representations of equations, students will see the equal sign as the midpoint or balance, with the quantity on the left of the equal sign is the same as the quantity on the right. When the quantities balance, there is **equality**. When there is an imbalance, there is **inequality**. The work in grade six extends this concept so that students discover that any change to one side must be matched with an equivalent change to the other side in order to maintain the balance. For example, if four is added to the left side of the equation, four must be added to the right side in order to preserve the equality.

In grades 3 and 4, **variables** are represented using a variety symbols such as circles and triangles. In grade 5, students were introduced to using letters as variables. However, students may have the misconception that $7w + 22 = 109$ and $7n + 22 = 109$ will have different solutions because the letter representing the variable has changed. Also they may see letters as objects rather than numerical values. Conventions of notations using variables may also produce misunderstandings. For example, $r \times z$ is written as rz, but 3×5 cannot be written as "35" and $2g$, where $g = 4$ means 2 times *4*, not 24.

When using variables, or representing variables using concrete objects, such as paper bags or boxes, students need to be directly taught that if the same variable, or object, is used repeatedly in the same equation, then there is only one possible solution for that variable or unknown.
For the example below, $c + c = 6$ or $2c = 6$ (c must represent the same number).





c must be worth 3

Students should explore equivalent forms of a given equation by applying the preservation of equality and verify using concrete materials on a balance. They should draw and record the original equation, then draw and record the results after adding the same amount to both sides, subtracting the same amount from both sides, multiplying both sides by the same factor, or dividing both sides by the same divisor.

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:

- Unit 1, Lesson 7, pp. 33-35
- Unit 1, Lesson 8, pp. 36-39

SCO: **6.PR3 Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

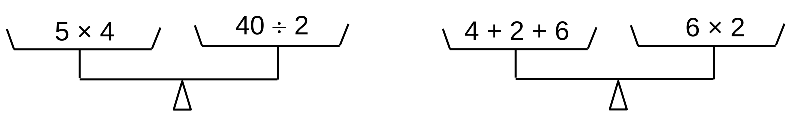
Instructional Strategies

Consider the following strategies when planning lessons:

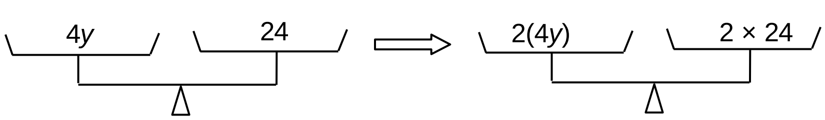
- Build known quantities on balance scales to model equality and also model how changes to one side must be matched with equivalent changes on the other. For example, model 3 cubes plus 5 cubes on one side and 8 cubes on the other. Have students record the equation. Then model adding 4 to both sides, subtracting 2 from both sides, doubling both sides, halving both sides, etc. Have the students record the equations.
- Model concrete examples of equations that include a variable, such as $3 + x = 10$. Model and record the preservation of equality when 5 is added to each side (e.g., $3 + x + 5 = 10 + 5$). Also explore the preservation of equality using subtraction, multiplication and division on both sides of the equation.
- Provide students with a number of equations such as $32 + 16 = k + 32$ and discuss how this can be solved without computation. Draw students' attention to the relationship of the left side to the right of the equation and that addition is not required to solve for the variable. Discuss the commutative property, changing the order of the **addends** or **factors** does not change the answer
- Use websites such as Learn Alberta to provide opportunities to further explore this concept: www.learnalberta.ca/content/mesg/html/math6web/lessonLauncher.html?lesson=m6lessonshell11.swf

Suggested Activities

- Extend the activity of "Tilt or Balance" game (Van de Walle & Lovin, vol. 3, 2006, p. 279) to include adding and subtracting variables.
- Provide a variety of illustrations of pan balances with expressions on each side. Ask students to determine if they balance.



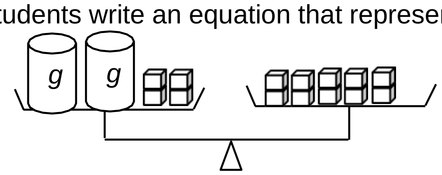
- Provide illustrations of pan balances that show equal expressions. Ask students to draw and record the shown equation, then draw and record the results when adding the same amount to both sides, subtracting the same amount from both sides, multiplying both sides by the same factor, and dividing both sides by the same divisor.

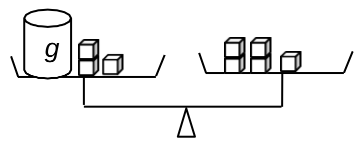


SCO: **6.PR3 Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

Assessment Strategies

- Tell students that x represents a certain number. Ask: Why must the solutions to $2x + 8 = 18$ and $2x + 4 = 14$ be the same?
- Ask students, "Which would have the larger value, "n" or "y"? Explain using numbers, pictures, and/or words.
  $2n + 6 = 14$ $16 = 2y + 6$
- Ask students to model the following equations using balance scales and various materials.
  Examples: $12 + 2s = 18$
  $17 = 5b - 3$
  $3p = 18 \div 2$
- Have students write an equation that represents each model:

a.


b.


Ask:
- Are the equations for these two scales equivalent? How do you know?
- Draw and record what will happen if you add 2 cubes to each side. Repeat for subtracting 2 from each side.
- Draw and record what will happen if you divide both sides of (a.) by 2.
- Draw and record what happens if you multiply both sides of (b.) by 3.

- Have students write two equations that are equivalent to $4m = 12$. Explain how they are equivalent.

SHAPE AND SPACE

SPECIFIC CURRICULUM OUTCOMES

Measurement

6.SS1 – Demonstrate an understanding of angles by: identifying examples of angles in the environment; classifying angles according to their measure; estimating the measure of angles using 45°, 90° and 180° as reference angles; determining angle measures in degrees; drawing and labeling angles when the measure is specified.

6.SS2 – Demonstrate that the sum of interior angles is: 180° in a triangle; 360° in a quadrilateral.

6.SS3 – Develop and apply a formula for determining the: perimeter of polygons; area of rectangles; volume of right rectangular prisms.

3D Objects and 2D Shapes

6.SS4 – Construct and compare triangles, including: scalene; isosceles; equilateral; right; obtuse; and acute in different orientations.

6.SS5 – Describe and compare the sides and angles of regular and irregular polygons.

SCO: **6.SS1 Demonstrate an understanding of angles by:**

- **identifying examples of angles in the environment**
- **classifying angles according to their measure**
- **estimating the measure of angles using 45°, 90° and 180° as reference angles**
- **determining angle measures in degrees**
- **drawing and labelling angles when the measure is specified.**

[C, CN, ME, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Five	Grade Six	Grade Seven
	6.SS1 Demonstrate an understanding of angles by: • identifying examples of angles in the environment • classifying angles according to their measure • estimating the measure of angles using 45°, 90 and 180° as reference angles • determining angle measures in degrees • drawing and labelling angles when the measure is specified.	**7.SS1** Demonstrate an understanding of circles by: describing the relationships among radius, diameter and circumference of circles; relating circumference to pi; determining the sum of the central angles; constructing circles with a given radius or diameter; solving problems involving the radii, diameters and circumferences of circles.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. provide examples of angles found in the environment;

B. classify a given set of angles according to their measure, e.g., acute, right, obtuse, straight, reflex;

C. sketch 45°, 90° and 180° angles without the use of a protractor, and describe the relationship among them;

D. estimate the measure of an angle using 45°, 90° and 180° as reference angles;

E. measure, using a protractor, given angles in various orientations;

F. draw and label a specified angle in various orientations using a protractor;

G. describe the measure of an angle as the measure of rotation of one of its sides; and

H. describe the measure of angles as the measure of an interior angle of a polygon.

SCO: **6.SS1 Demonstrate an understanding of angles by:**
- **identifying examples of angles in the environment**
- **classifying angles according to their measure**
- **estimating the measure of angles using 45°, 90° and 180° as reference angles**
- **determining angle measures in degrees**
- **drawing and labelling angles when the measure is specified.**

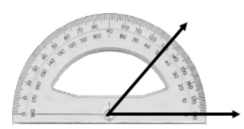
[C, CN, ME, V]

Elaboration

Students have been previously introduced to the idea of angles during their study of polygons but in Grade 6 the properties angles are explored in greater depth. Frequently, angles are defined as the meeting of two **rays** at a common **vertex**. It is more useful, however, for students to conceptualize an angle as a **turn** and the measure of the angle as the amount of turn. It is important for students to understand that:
- a larger angle corresponds to a greater turn from the starting position
- the length of the arms (rays) of the angle does not affect the turn amount and, therefore, does not affect angle size
- the orientation of an angle does not affect its measurement or classification.

It is also important that students learn the different types of angles and be able to **classify** them as **acute** (less than 90°), **right** (exactly 90°), **obtuse** (greater than 90° and less than 180°), **straight** (exactly 180°), **reflex** (more than 180°).

Students should learn how to use a **protractor** to measure angles accurately. When drawing or measuring angles, students need to be reminded that the centre point of the protractor needs to be lined up with the vertex of the angle, and the 0° line of the protractor must line up exactly with one ray of the angle. Students typically use protractors with double scales and will need to learn how to determine which set of numbers to use in a given situation. This is best accomplished by first having the student estimate the size of the angle with known benchmark angles such as 45°, 90° and 180° and then decide which reading makes the most sense. For example, the angle shown below is obviously an acute angle, and therefore its measure is 50°, not 130°.



This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:
- Unit 4, Lesson 1, pp. 126-129
- Unit 4, Lesson 2, pp. 130-132
- Unit 4, Lesson 3, pp. 133-138
- Unit 4, Lesson 4, pp. 139-142
- Unit 4, Game, p. 143
- Unit 4, Lesson 5, pp. 144, 145
- Unit 4, Unit Problem, pp. 156, 157

SCO: **6.SS1 Demonstrate an understanding of angles by:**
- **identifying examples of angles in the environment**
- **classifying angles according to their measure**
- **estimating the measure of angles using 45°, 90° and 180° as reference angles**
- **determining angle measures in degrees**
- **drawing and labelling angles when the measure is specified.**

[C, CN, ME, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Present angles in a variety of real life contexts (e.g., angles formed by the two hands of a clock, by the intersection of two roads, and by the blades of scissors or hedge clippers).
- Explore the similarities between rulers and protractors. Students should recognize that protractors work similarly to rulers and the distance between the two rays is what should be counted (measured).
- Show students angles (with arms of different lengths) in various positions and of different sizes. Ask them to estimate each (e.g., almost 45°, 90°, 180°, etc.).
- Have students find angles in various 2D polygons and on faces of 3D object. Students could use any right-angled (90°) corner of a piece of paper to check their estimates. Folding this corner in half could also help visualize half a right angle (45°).
- Have students stand with their arms closed on top of each other pointing out in the same direction to the side. This shows (0°)**.** Then have them raise one arm up until it points directly up (90°), then continue rotating their arm until their arms are out straight to make straight angles (180°).
- Have students create their own non-standard unit protractors. Provide the students with semicircular shapes cut from tracing paper, or waxed paper. Have them fold the semicircle in half, forming a right angle or square corner. Explain that angles are measured in degrees and that a right angle is 90 degrees. Ask them to fold once again and determine and name the new angles created by the folds. Discuss the measurement of these folds and how they can assist with estimation of angle sizes.

Suggested Activities

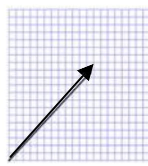
- Have students investigate angles in various shapes, using the corner of a piece of paper as a reference for right angle. Does it fit the angle of the shape or is the angle greater/less than the corner of the paper?
- Have students make various angles with pipe cleaners or geo-strips (e.g., almost a right angle, about 45°, a right angle, a straight angle, a reflex angle).
- Ask students to explore the angles in the six different pattern blocks. Which blocks have only acute angles? Only obtuse angles? Both acute and obtuse angles? Only right angles? Reflex angles?
- Display different times one at a time on overhead clocks. Ask students to name and describe the angle made by the hands.
- Ask students to measure the angles found in various letters of the alphabet.
- Ask students where acute, right, obtuse, straight, and reflex angles could be identified in the classroom.

SCO: **6.SS1 Demonstrate an understanding of angles by:**
- **identifying examples of angles in the environment**
- **classifying angles according to their measure**
- **estimating the measure of angles using 45°, 90° and 180° as reference angles**
- **determining angle measures in degrees**
- **drawing and labelling angles when the measure is specified.**

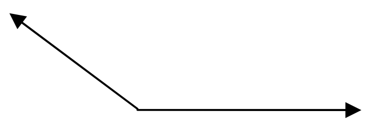
[C, CN, ME, V]

Assessment Strategies

- Ask students to combine two or more pattern blocks to make examples of acute, right, straight, and obtuse angles. Have them record by tracing each one on paper.
- Tell students that the hands of a clock are forming a given angle (such as 45°). Ask what time it could be.
- Show the student the diagram below and ask why it is easy to tell that it is 45°.



- Show students an angle of, for example 135°, and tell them that someone said that it was 45°. Ask students to explain how he/she thinks such an error could be made.
- Provide students with various angles to measure.
- Have students draw angles with specified measures.
- Ask students how can you use a 90° angle to construct a 45° angle?
- Tell students that Trevor measured the angle below and said it measured 50°. What was his error?



SCO: **6.SS2 Demonstrate that the sum of interior angles is:**
- **180° in a triangle**
- **360° in a quadrilateral.**

[C, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.SS4 Identify and sort quadrilaterals, including: rectangles; squares; trapezoids; parallelograms; rhombuses according to their attributes.	**6.SS2** Demonstrate that the sum of interior angles is: • 180° in a triangle • 360° in a quadrilateral.	**7.SS1** Demonstrate an understanding of circles by: describing the relationships among radius, diameter and circumference of circles; relating circumference to pi; determining the sum of the central angles; constructing circles with a given radius or diameter; solving problems involving the radii, diameters and circumferences of circles.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. explain, using models, that the sum of the interior angles of a triangle is the same for all triangles; and

B. explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.

SCO: **6.SS2 Demonstrate that the sum of interior angles is:**
- **180° in a triangle**
- **360° in a quadrilateral.**

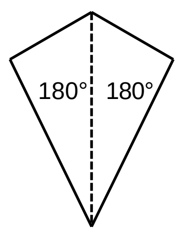
[C, R]

Elaboration

In previous grades, students have experienced the attributes of polygons, but this has been limited to side lengths and other properties of the sides. They will have informally explored angles and will build on these experiences as angles are investigated in greater depth in grade 6. It is recommended that 6.SS1 and 6.SS4 be taught before this outcome, so that students are familiar with the measurement of angles, the different types of triangles, and the vocabulary to name and describe them.

Through explorations, students should discover that the angles of a triangle add to 180**°**. This can be done using paper models and/or dynamic geometry software such as *Geometer's Sketchpad*, Smart Board *Notebook*, or interactive websites such as GeoGebra. Different types of triangles (acute-angled, isosceles, obtuse-angled, equilateral, etc.) need to be used so that students discover that this property applies to all types of triangles.

Exploration of the angle properties of triangles should be extended to **quadrilaterals** by concretely investigating the relationship between triangles and quadrilaterals. Students should discover that two triangles can be combined to create a quadrilateral and conclude that the sum of the angles is 360**°** (180° + 180°).



This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:
- Unit 4, Lesson 6, pp. 146-149
- Unit 4, Lesson 7, pp. 150-153
- Unit 4, Unit Problem, pp. 156, 157

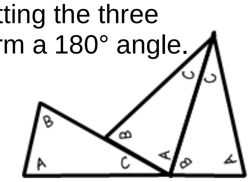
SCO: **6.SS2 Demonstrate that the sum of interior angles is:**
- **180° in a triangle**
- **360° in a quadrilateral.**

[C, R]

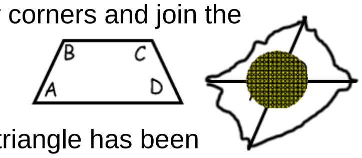
Instructional Strategies

Consider the following strategies when planning lessons:

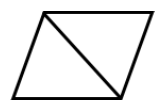
- Have students draw a triangle of any type and label its angles 1, 2, 3. Cut it out. Then have the student tear off the three angles and place the three vertices together to form a 180° angle. Have students measure and record the three angles and add them.
- Use graphic software and create three congruent triangles. Rotate as described above.
- Have students cut out three congruent triangles by stacking three sheets of paper and cutting the three shapes at once. Rotate the triangles so the three different vertices meet at one point to form a 180° angle.



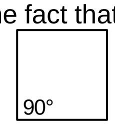
- Cut out a quadrilateral and label the four vertices. Have students tear off the four corners and join the vertices together. Highlight the 360° sum.



- Have students draw and cut out a quadrilateral once the sum of the angles of a triangle has been explored and determined. Have them determine that a quadrilateral can be made up of two triangles, and the sum of the angles of those two triangles equals 360°.



- Explore how the characteristics of a square are helpful for students to remember the fact that every quadrilateral has a sum of angles equal to 360°.



Suggested Activities

- Have students each draw a variety of different triangles. Have them measure, record, and add the angles of each one. Have them discuss their findings until they reach the conclusion that the sum of the angles of *any* triangle is 180°. Repeat the above activity using a variety of quadrilaterals.
- Provide a variety of triangles with the measures of two angles shown. Students must find the measure of the third angle using their understanding of the sum of the angles of a triangle (without a protractor).
- Ask students to predict the interior angle of an equilateral triangle, and then check by measuring with a protractor.
- Provide students with a variety of quadrilaterals with the measures of three of the angles given. Students must find the measure of the fourth angle without a protractor.
- Ask students to predict the interior angle of an equilateral triangle, and then check by measuring with a protractor.

SCO: **6.SS2 Demonstrate that the sum of interior angles is:**
- **180° in a triangle**
- **360° in a quadrilateral.**

[C, R]

Assessment Strategies

- Ask students can a triangle have more than one obtuse angle? Why or why not? Explain using numbers, pictures, and/or words.
- Ask students can a triangle have two right angles? Why or why not? Explain using numbers, pictures, and/or words.
- Tell students that any quadrilateral can be divided into two triangles. Since the sum of the angles on one triangle is 180°, it is obvious that the sum of the angles of a quadrilateral must be 360°. Explain what you think about the statement using numbers, pictures, and/or words.
- Have students solve to find the measure of the third angle of a triangle when the measures of the other two angles are given.
- Have students solve to find the measure of the fourth angle of a quadrilateral when the measures of the other three angles are given.

SCO: **6.SS3 Develop and apply a formula for determining the:**
- **perimeter of polygons**
- **area of rectangles**
- **volume of right rectangular prisms.**

[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.SS1 Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and draw conclusions. **5.SS3** Demonstrate an understanding of volume by: selecting and justifying referents for cm^3 or m^3 units; estimating volume by using referents for cm^3 or m^3; measuring and recording volume (cm^3 or m^3) constructing rectangular prisms for a given volume.	**6.SS3** Develop and apply a formula for determining the: • perimeter of polygons • area of rectangles • volume of right rectangular prisms.	**7.SS2** Develop and apply a formula for determining the area of: parallelograms; triangles; circles.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. explain, using models, how the perimeter of any polygon can be determined;
B. generalize a rule (formula) for determining the perimeter of polygons, including rectangles and squares;
C. explain, using models, how the area of any rectangle can be determined;
D. generalize a rule (formula) for determining the area of rectangles;
E. explain, using models, how the volume of any right rectangular prism can be determined;
F. generalize a rule (formula) for determining the volume of right rectangular prisms; and
G. solve a given problem involving the perimeter of polygons, the area of rectangles and/or the volume of right rectangular prisms.

SCO: **6.SS3 Develop and apply a formula for determining the:**
- **perimeter of polygons**
- **area of rectangles**
- **volume of right rectangular prisms.**

[C, CN, PS, R, V]

Elaboration

The basic concepts of perimeter, area, and volume have been introduced and explored in previous grades. Students have estimated and worked with both non-standard and standard units. The emphasis for grade 6 is to have students discover the *most efficient strategies* for finding these measures. By exploring the number relationships and patterns of perimeters and areas students should make generalizations. These explorations should eventually elicit from students the traditional **formulas** for **perimeter of polygons**, **area of rectangles**, and **volume of right rectangular prisms**.

As a result of prior experiences, students should conceptualize perimeter as the total distance around an closed object or figure. They might observe that, for certain polygons, the perimeter is particularly easy to compute.
- Equilateral triangle: the perimeter is 3 times the side length.
- Square: the perimeter is 4 times the side length.
- Rectangle: the perimeter is double the sum of the length and the width.

Students will be familiar with the concept of area from grade 4, where they found the area of rectangles using standard units. “From earlier work with multiplication and the array meaning or model of multiplication, students will know that, to determine the total number of squares, you multiply the number of rows of squares by the number of squares in each row” (Small, 2008, p. 398). Students need to have many opportunities to experiment with the relationships among length, width, and area to develop their own formulas for area of rectangles (remind students that a square is a special type of rectangle).

Volume has been studied in grade 5. Students should recognize volume as:
- the amount of space taken up by a 3-D object; or
- the amount of cubic units required to build and fill the object.

Students should also recognize that each of the three dimensions of the prism affects the volume of the object. Development of the concept of using the **area of the base** as part of the formula for volume of a right rectangular prism will be helpful for work in later grades as volume of other 3D objects is explored.

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:
- Unit 6, Lesson 7, pp. 226-230
- Unit 6, Lesson 8, pp. 231-234
- Unit 6, Lesson 9, pp. 235-238
- Unit 6, Game, p. 239
- Unit 6, Unit Problem, pp. 242, 243

SCO: **6.SS3 Develop and apply a formula for determining the:**
- **perimeter of polygons**
- **area of rectangles**
- **volume of right rectangular prisms.**

[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students examine the perimeter of regular polygons with various side lengths. They could record the data for a regular hexagon as shown in the table below.

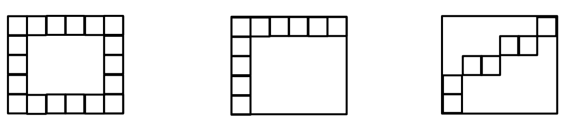
Side length (cm)	1	2	3	4	5	6
Perimeter (cm)	6	12	18	24	30	36

The next step is to have students generalize the pattern they have found for the perimeter of regular hexagons, stating the pattern rule as an algebraic equation: P(hexagon) = 6*n*
Other types of generalizations can be developed through measurement and pattern tables as students explore the perimeters of other regular polygons and areas of rectangles.

- Provide pictures of many regular polygons, with the measure of one side provided for each. Have students explore to find the most efficient method for finding the perimeters of each. Lead students to discover that “side + side + side + side…” is inefficient when multiplication can be used instead. Repeat the activity with rectangles and parallelograms.
- Provide students with graphics of many rectangles, including squares, in which the square units are shown and the length and width measures are given. Ask students to find the most efficient way to find the areas of each. Begin with small areas, such as 2 cm × 3 cm, and help students relate these rectangles to the array model of multiplication.
- Have students create many different rectangles, including squares, on grid paper. Have them find and record the length, width, and area for each (by counting the squares, if necessary). They should record their findings in chart form, so they can look for relationships in the table among the length, width, and area for each. Lead students to develop the formula: *length* × *width* (Small, 2008, p. 398).
- Have students build a variety of right rectangular prisms. On a chart, have them record the length and width of the base and the height, as well as the volume. Have students look for relationships among these measures and lead students to developing the formula

Suggested Activities

- Provide students with a variety of rectangles with incomplete grids. Have them apply the formula to determine their areas.



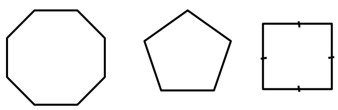
- Provide 3D regular polygon objects to explore to find patterns between side lengths and create a rule (formula) for each to calculate the perimeter.
- Present students with rectangular prisms constructed out of linking cubes. Have them calculate the volume. Determine if the student uses multiplication rather than counting cubes.
- Provide students with linking cubes and have them build cubes of different sizes. In each case, ask them to record the various side lengths and volumes in a table. Then ask them to predict the volume of a cube with a side length of 2.5 units.

SCO: **6.SS3 Develop and apply a formula for determining the:**
- **perimeter of polygons**
- **area of rectangles**
- **volume of right rectangular prisms.**

[C, CN, PS, R, V]

Assessment Strategies

- Tell students that the perimeter of a triangle is 15 cm. Have them describe and draw the possible side lengths. (Note: if outcome **6.**SS4 has been done, the type of triangle can be specified – scalene, isosceles, etc.)
- Ask students: "How can a formula be used to determine the perimeter of the following regular polygons?"



- Provide students with problems to solve such as the following.
  - "A teen mowed two lawns. One lawn was 10 m by 12 m, and the other was 15 m by 10 m. The teen charges $3.00 for each 10 m^2. How much was charged for the two lawns?"
- Provide students with the dimensions of a real world container that is a rectangular prism (e.g., a carton, a box, a popcorn bag, etc.). Ask students to find the perimeter and area of each face. Students should also determine the volume for the prism. Ask students to determine the possible dimensions if the object needed to hold twice as much.
- Have students explain, using numbers, pictures, and/or words, why a rectangular prism that is 5 cm by 3 cm, with a height of 4 cm must have a volume of 60 cm^3.

SCO: **6.SS4 Construct and compare triangles, including:**
- **scalene**
- **isosceles**
- **equilateral**
- **right**
- **obtuse**
- **acute**

in different orientations.
[C, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Five	Grade Six	Grade Seven
	6.SS4 Construct and compare triangles, including: • scalene • isosceles • equilateral • right • obtuse • acute in different orientations.	

Achievement Indicators

Students who have achieved this outcome should be able to:

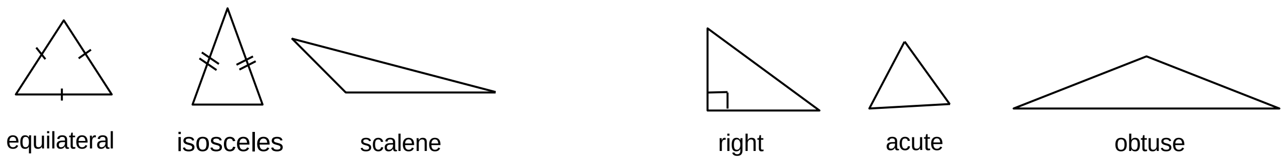
A. sort a given set of triangles according to the length of the sides;
B. sort a given set of triangles according to the measures of the interior angles;
C. identify the characteristics of a given set of triangles according to their sides and/or their interior angles;
D. sort a given set of triangles and explain the sorting rule;
E. draw a specified triangle, e.g., scalene; and
F. replicate a given triangle in a different orientation and show that the two are congruent.

SCO: **6.SS4 Construct and compare triangles, including:**
- **scalene**
- **isosceles**
- **equilateral**
- **right**
- **obtuse**
- **acute**

in different orientations.
[C, PS, R, V]

Elaboration

Students need to realize triangles can be sorted either by the length of their sides (**equilateral**, **isosceles**, **scalene**) or by the size of their angles (**right**, **acute**, **obtuse**).



Students should explore through discussion why there are only three possible classifications by side length. It should be discovered that triangles cannot be classified according to one "equal side", but there needs to be zero, two, or three equal sides. Similar discussion may be held around the reasons for the three different types of triangles in the set of classifications by angle size. For example, a triangle cannot have more than one obtuse angle (greater than 90°) as the angles in a triangle add up to 180°. Once these two sets of classification have been studied, teachers should extend students' knowledge to explore how a triangle may fall into two categories at the same time (e.g., a right scalene triangle, an obtuse isosceles triangle, etc.).

Students have not used the term **congruent** before this point, although they have had experience comparing and matching 2D shapes based on attributes. It would be helpful to introduce the symbol for congruent ($\cong$).

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:
- Unit 6, Lesson 1, pp. 200-204
- Unit 6, Lesson 2, pp. 205-208
- Unit 6, Lesson 3, pp. 209-213
- Unit 6, Unit Problem, pp. 242, 243

SCO: **6.SS4 Construct and compare triangles, including:**
- **scalene**
- **isosceles**
- **equilateral**
- **right**
- **obtuse**
- **acute**

in different orientations.
[C, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Explore triangles by folding to compare and measuring the angles could lead to discovering these patterns: a) all angles in equilateral triangles are equal; b) two angles in isosceles triangles are equal; and c) all angles in scalene triangles are different.
- Have students test for congruency by placing one shape on top of each other to see if the outlines match exactly.
- Give students cards with examples of right, acute, and obtuse triangles on them. Ask them to sort them into three groups by the nature of their angles and share how they were sorted. Attach the names for these classifications to the students' groups.
- Use Venn diagrams or Carroll diagrams as a useful graphic organizer for sorting triangles.

Suggested Activities

- Prepare pictures on cards or cutouts of several examples of these three kinds of triangles. Ask students to sort them into three groups. Ask them to explain their sort. Often, they will sort them by how their sides look, without knowing the actual names. If so, this will lead to a focus on measuring and comparing the sides, and noting common properties to which the names equilateral, isosceles, and scalene can be attached. (If not, the teacher may sort them, ask students to determine the sorting rule, and do other explorations.)
- Identify everyday examples of each type of triangle; yield sign, bridges, the side of a Toblerone bar, other support items, ladder against a wall. Students should also examine familiar materials in the classroom, such as pattern blocks and tangrams.
- Provide pairs of students with two 6 cm straws, two 8 cm straws, and two 10 cm straws. Have them investigate the triangles they can make using 3 straws at a time and complete a table with their results. This activity could be varied by using toothpicks or geo-strips.

Straws Used	Type of Triangle

- Provide students with paper cut-outs of various types of triangles. Have them explore how many different orientations of the same triangle they can find and trace.
- Have students draw a triangle on tracing paper and classify it. Have them fold the paper in order to trace the shape several different ways to create congruent triangles in other orientations.

SCO: **6.SS4 Construct and compare triangles, including:**
- **scalene**
- **isosceles**
- **equilateral**
- **right**
- **obtuse**
- **acute**

in different orientations.
[C, PS, R, V]

Assessment Strategies

- Have students draw a scalene right triangle, an isosceles, acute-angle triangle, and other examples of combined classifications.
- Have students construct specific triangles on their geoboards and record them on dot paper (e.g., an acute triangle that has one side using five pins; a right triangle that is also isosceles; an obtuse triangle that has one side using five pins).
- Have students draw a specified triangle type, such as:
  a. an obtuse triangle with an angle of 130°.
  b. a triangle with 3 cm and 4 cm sides that form a right angle.
  c. an equilateral triangle with 10 cm sides.
  d. an obtuse triangle with a 110° angle and one 5 cm side .
- Tell students that one side of a triangle is 20 cm. What might the lengths of the other two sides be for each of the followings kinds of triangles?
  - isosceles
  - scalene
  - equilateral

SCO: **6.SS5 Describe and compare the sides and angles of regular and irregular polygons.**
[C, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.SS4 Identify and sort quadrilaterals, including: rectangles; squares; trapezoids; parallelograms; rhombuses according to their attributes.	**6.SS5** Describe and compare the sides and angles of regular and irregular polygons.	

Achievement Indicators

Students who have achieved this outcome should be able to:

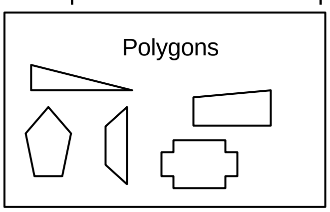
A. sort a given set of 2D shapes into polygons and non-polygons, and explain the sorting rule;
B. demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing;
C. demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring;
D. demonstrate that the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure;
E. sort a given set of polygons as regular or irregular and justify the sorting; and
F. identify and describe regular and irregular polygons in the environment.

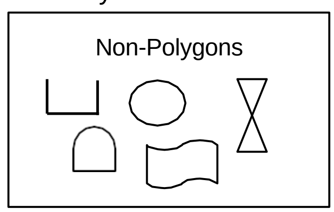
SCO: **6.SS5 Describe and compare the sides and angles of regular and irregular polygons.**
[C, PS, R, V]

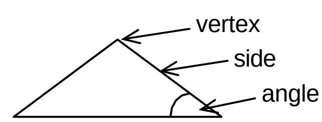
Elaboration

Initially, students identified shapes by their overall appearance emphasizing the properties of the sides. While other properties have been informally explored, the focus of grade 6 is to include all of the **properties** of **sides** and **angles**. Teachers need to provide students with questions and opportunities to guide their investigations of 2D shapes through sorting activities.

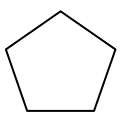
Polygons are closed 2D shapes with three or more straight sides. The sides intersect only at the **vertices**. A key property of polygons is that the number of sides is always equal to the number of vertices. Shapes that are missing one or more of these attributes are considered **non-polygons**. It is important that students focus on these attributes to determine whether the shape is a polygon. A common misconception is to think that triangles and quadrilaterals are not polygons since they have other names.

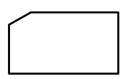






In grade 6, students will extend their knowledge to include both regular and irregular polygons. **Regular polygons** have all sides and angles equal (e.g., equilateral triangles, squares, yellow hexagon pattern blocks). **Irregular polygons** do not have all sides nor angles that are the same size. Students should be given opportunities to explore both regular and irregular polygons in their environment. Using the attributes of polygons, students should be able to sort into regular or irregular polygons.


Regular pentagon


Irregular pentagon

It is also important for students to investigate the concept of congruence through direct comparison (laying one shape on top of the other) and by measuring the sides and angles.

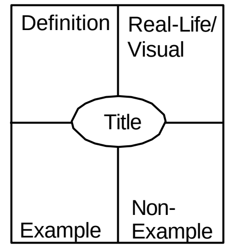
This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:

- Unit 6, Lesson 4, pp. 214-218
- Unit 6, Lesson 5, pp. 219-223
- Unit 6, Lesson 6, pp. 224, 225
- Unit 6, Unit Problem, pp. 242, 243

SCO: **6.SS5 Describe and compare the sides and angles of regular and irregular polygons.**
[C, PS, R, V]

Instructional Strategies

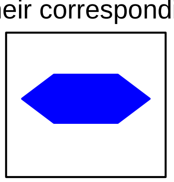
Consider the following strategies when planning lessons:



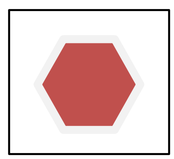
- Provide students with a template for the Frayer Model and have them fill in the sections, individually or as a group, to consolidate their understanding of the properties of polygons and non-polygons. This activity may be repeated to distinguish the attributes of regular and irregular polygons.
- Have students prepare property lists with headings: sides, angles. Using a collection of regular and irregular polygons (models or pictures on cards), have students describe the shapes using language such as: all sides equal, 2 angles the same, opposite sides equal, no sides equal, etc. Then have the students sort the polygons into regular or irregular polygons.
- Provide students with a list of attributes and have them construct a polygon that has the set of attributes. Have students share and compare with the class.
- Display models or copies of regular polygons on the board. Place a smaller version of the regular polygon on the overhead projector. Have students move the projector until the image matches, with the one taped on the board. This will help to prove the congruence of their angles, regardless of their side lengths. Interactive white boards can also be an effective tool to show congruency of angles of regular polygons.

Suggested Activities

- Have students work in pairs to prepare a concentration card game with pictures of regular and irregular polygon and their corresponding names.



irregular hexagon



regular hexagon

- Have students trace a regular polygon (e.g., yellow pattern block). Have them rotate their shape to prove the congruency of sides and angles. The congruency should be double-checked by measuring the polygon's angles and sides.
- Have students go on a "Polygon and Irregular Polygon Scavenger Hunt". Have them sort their polygons with similar attributes and explain their rules for sorting.
- Provide several copies of a non-regular polygon that has been rotated and reflected a number of different ways. Have students cut out one shape and superimpose it over the others to prove congruency. This can be done using paper drawings or on the computer. Incongruent shapes may be included. Extend this activity by having students repeat the activity with a shape of their own.

SCO: **6.SS5 Describe and compare the sides and angles of regular and irregular polygons.**
[C, PS, R, V]

Assessment Strategies

- Provide a set of many polygons. Have students match the congruent pairs.
- Have students create a T-chart with the headings of polygons and non-polygons. Then fill in chart with shapes that could include shapes from their environment (e.g., oval, pentagon, tiles, rectangular window, face of a clock, triangle seen on a roof, angle, sheet of paper, etc.). The task could be repeated with having students identify regular and irregular polygons.
- Provide students with several different polygons (regular and irregular) to sort and have them justify their sorting rule.
- Have students draw congruent polygons that satisfy a given set of attributes. Students should be able to prove the shapes are congruent by measuring.
- Provide two congruent irregular polygons. Have students prove congruency by measuring and labelling the sides and angles.

STATISTICS AND PROBABILITY

SPECIFIC CURRICULUM OUTCOMES

Data Analysis

6.SP1 – Create, label and interpret line graphs to draw conclusions.

6.SP2 – Graph collected data and analyze the graph to solve problems.

Chance and Uncertainty

6.SP3 – Demonstrate an understanding of probability by: identifying all possible outcomes of a probability experiment; differentiating between experimental and theoretical probability; determining the theoretical probability of outcomes in a probability experiment; determining the experimental probability of outcomes in a probability experiment; comparing experimental results with the theoretical probability for an experiment

SCO: **6.SP1 Create, label and interpret line graphs to draw conclusions.**
[C, CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.SP2 Construct and interpret double bar graphs to draw conclusions.	**6.SP1** Create, label and interpret line graphs to draw conclusions.	

Achievement Indicators

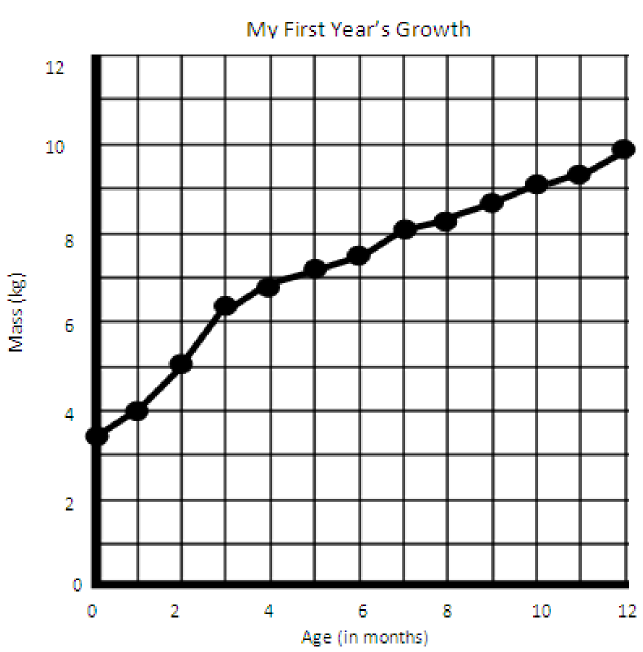
Students who have achieved this outcome should be able to:

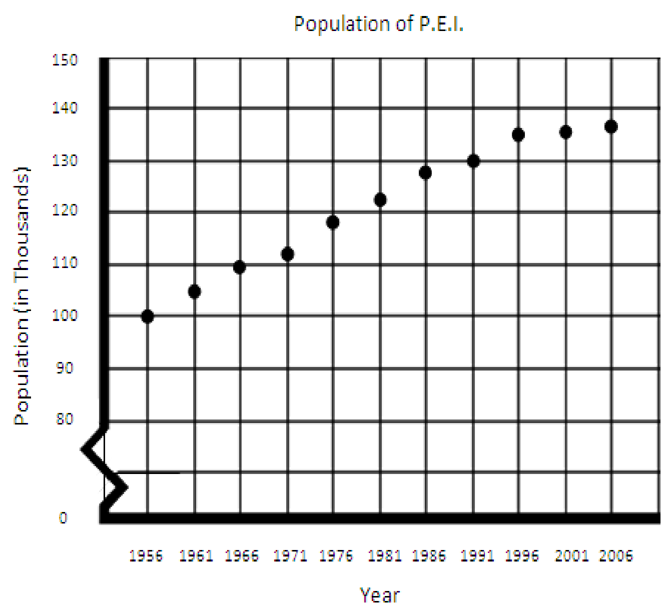
A. determine the common attributes (title, axes and intervals) of line graphs by comparing a given set of line graphs;

B. determine whether a given set of data can be represented by a line graph (continuous data) or a series of points (discrete data) and explain why;

C. create a line graph from a given table of values or set of data; and

D. interpret a given line graph to draw conclusions.





SCO: **6.SP1 Create, label and interpret line graphs to draw conclusions.**
[C, CN, PS, R, V]

Elaboration

Statistical literacy is a life skill that effective citizens use to read, question, and interpret data in our world. Graphs tell a story, so both the creator and the reader of the graph need to think about the message that is being shared.

Students have investigated about tables of values and describing patterns and relationships using graphs and tables in outcomes **6.**PR1 and **6.**PR2.

The points on a line graph are plotted to show relationships between two variables. The distinction between **continuous** and **discrete** data should be emphasized as students investigate line graphs. **Continuous data** includes an infinite number of values between two points and is shown by joining the data points with a straight line making it easier to focus on trends implicit in the data. Points on the line between the plotted points have meaning. **Discrete data** has a finite number of values (i.e., data that can be counted such as the number of pets), and therefore, the points in the graph should not be connected.

If the plotted points are not joined by straight lines because they represent discrete data, they may be called broken-line graphs or a series of plotted points. Every point on the line should have a value, thus a line graph can also be used to show values between points on the graph. Students should be able to determine the value of the plotted data points and those points between points.

Line graphs should include **a title, labelled categories (including units), labelled axes, an appropriate scale** and **correctly plotted points**. It is suggested that grade six students follow this criteria when creating a graph, although there is some variation in creation of graphs when graphs from the internet, newspapers, and/or books are examined. Be sure to expose students to a variety of graphs; this exposure will encourage thinking and discussion that will promote flexibility in one's mathematical thinking.

The purpose of a line graph is to focus on trends implicit in the data. For example, if students measure the temperature outside every hour during a school day, they could create a graph in which the ordered pairs (hour, temperature) are plotted. By connecting the points with line segments, they see the trend in the temperature. If a graph is not showing all the numbers in the scale, then a jagged line can be used to indicate the omission of some values. Ensure that the construction of the line graph and interpretation of the data are not addressed independently; when students take the time to construct line graphs, they should be used for interpretation.

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:

- Unit 7, Lesson 3, pp. 259-262
- Unit 7, Lesson 4, pp. 263-266

SCO: **6.SP1 Create, label and interpret line graphs to draw conclusions.**
[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure students are aware of the parts of line graphs (i.e. titles, labels, scales, etc.) using real graphs that are interesting to students.
- Introduce this concept with tables of values or sets of data to create line graphs, starting with plotting points and then connecting them.
- Provide real-world line graphs and ask questions that require students to read and interpret the information found there.
- Integrate the use of technology to construct line graphs. It is important that students also have the experience of creating graphs with paper-and-pencil methods.
- Use websites such as Statistics Canada or Elections Canada for up-to-date data and related resources.

Suggested Activities

- Have students collect information about the number of students in the school in Grades 1, 2, 3, 4, and 5 and draw a line plot to help show whether there are differences in the number of students in certain grades. Remind students to carefully consider the step size for the vertical scale.
- Have students record the changes in temperature over time during the day/week and create an appropriate line graph and label the title, axes, and scales.
- Ask students to look up the hockey scores for a favourite team over the course of 10 games and then create a line graph with the ordered pairs (game number, number of goals scored by favourite team). Have them create a second graph with the ordered pairs (game number, goals scored by opposing team) and then compare the two graphs.
- Using discrete data, ask groups of students to use the data to create either a broken line graph (also called a series of plotted points) or a bar graph. Discuss with students that the both types of graphs may be chosen to represent the information. Have students examine both types of graphs. Which graph provides the better message to the reader? Which graph has more visual impact? When might you choose a bar graph rather than a broken-line graph (series of points)? When might you choose a broken-line graph (series of points) rather than a bar graph?
- Refer to the graph in <u>*Science & Technology: Space*</u> on page 26. Does this graph represent continuous data or discrete data? How do you know? Discuss why they chose to use plotted points rather than a bar graph for this discrete data.

SCO: **6.SP1 Create, label and interpret line graphs to draw conclusions.**
[C, CN, PS, R, V]

Assessment Strategies

- Provide students with two line graphs displaying similar data (such as temperature change over time in two different areas) and have students write comparison statements based on the data shown.
- Have students create a line graph based on the following information using appropriate scales, labels, and title.

Number of Cups	1	2	3	4
Capacity (mL)	250	500	750	1000

- Have students explain (in words or pictures) the difference between continuous and discrete data.
- Provide an example of a line graph. Have students create three questions which can be answered from the graph.
- Have students explain three situations where a line graph would be appropriate to use.
- Provide a broken-line graph and have students explain why line graphs are not always linear.
- Have students create a line graph based on the table below. Have them determine about how much rain fell by 5:30 p.m. If the rain continues to fall steadily at the same rate, about how much will fall by 8:00 p.m.?

Time of Day	2:00 p.m.	3:00 p.m.	4:00 p.m.	5:00 p.m.	6:00 p.m.
Total rainfall	3 mm	5 mm	7 mm	9 mm	11 mm

SCO: **6.SP2 Graph collected data and analyze the graph to solve problems.**
[C, CN, PS]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Grade Five	Grade Six	Grade Seven
5.SP2 Construct and interpret double bar graphs to draw conclusions.	**6.SP2** Graph collected data and analyze the graph to solve problems.	

Achievement Indicators

Students who have achieved this outcome should be able to:

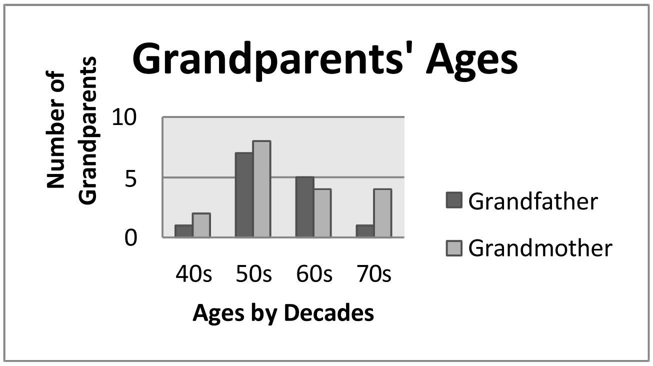
A. determine an appropriate type of graph for displaying a set of collected data and justify the choice of graph; and

B. solve a given problem by graphing data and interpreting the resulting graph.

SCO: **6.SP2 Graph collected data and analyze the graph to solve problems.**
[C, CN, PS]

Elaboration

Students should regularly use pictographs, bar graphs, double bar graphs, and line graphs to display and organize data. Data can be collected in surveys, through experiments or through research. Topics may include areas of mathematics, other curricular areas and real-life situations. For example, students might gather information about the ages of their grandparents and display it in various types of graphs.



Grandparents' Ages

Grandparents		
In their 40s:	Grandfather Grandmother	☺ ☺ ☺
In their 50s:	Grandfather Grandmother	☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺
In their 60s:	Grandfather Grandmother	☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺
In their 70s:	Grandfather Grandmother	☺ ☺ ☺ ☺ ☺
☺ = 1 grandparent		

Students should recognize that a line graph would not be appropriate for information, since the data is not continuous. It is a count of the number of grandparents in each age range.

When creating graphs, students need to include a title, labelled categories (including units), labelled axis, an appropriate scale and correctly plotted points, bars or pictures. If a legend or key is needed, then students need to provide that information on their graph.

It is also important for students to explore the various types of data displays (line, bar, pictograph) and how these different displays are not always equally effective or appropriate. Students may wish to explore circle graphs as they are a common type of data display.

Students need to realize that data should be collected to answer questions and solve relevant problems. One often wishes to gather information about a large population, but does not have the ability to check every person involved. In situations such as these, samples are used. One then generalizes to the entire population, recognizing that conclusions drawn from the sample may not be perfectly true for the entire group, but trying to choose the sample to minimize the degree of error.

Students should consider both how to choose samples and how safe it is to generalize to the full populations. For example, suppose one wanted to determine people's favourite take-out food. It would not be wise to choose a sample of patrons of Pizza Palace. Clearly, that sample could be biased in favour of pizza.

In choosing a sample, students should carefully consider the information being sought and how a person answering (a) question(s) could be biased. For example, if students want to find what radio station is most popular, they should probably consider: the mix of ages within the sample; the gender distribution within the sample; the availability of a variety of stations to those sampled; the time of day (some stations are likely more attractive to listeners at a particular time of day). A sample should be constructed to deal with such potential biases.

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:

- Unit 7, Lesson 1, pp. 248-251
- Unit 7, Lesson 4, pp. 263-266
- Unit 7, Lesson 5, pp. 267-270
- Unit 7, Unit Problem, pp. 286, 287

SCO: **6.SP2 Graph collected data and analyze the graph to solve problems.**
[C, CN, PS]

Instructional Strategies

Consider the following strategies when planning lessons:

- Collect data as a class or individually. Have students to place the data in a table, and choose an appropriate graph to display it. Ask students to explain their reasoning for their choice of graph.
- Use the Internet as a source of data and possible lesson ideas such as:
  - Statistics Canada Statistics Canada
  - World Fact Book
  - Internet Movie Database
- Examine many real-world pictographs, bar, double-bar, and line graphs gathered from newspapers, magazines, and other print media. Discuss why the choice of format is appropriate in each case. Ask students questions that can be answered through careful analysis of the graph.
- Provide meaningful questions that students can answer by gathering and graphing data. Examples:
  - We're having a sock hop and need to choose some CD's. What are the most popular styles of music?
  - What were the most frequently observed types of insects from our Science "Variety of Life" activity?
  - If we order t-shirts for our school, what are the most popular sizes we need to get?
  - What are the distances our paper airplanes travelled in our "Flight" experiment?
  - We need to track the number of apples sold at our canteen over the past 6 weeks so we'll know how many more to order.

Suggested Activities

- Use these ideas as classroom questions to collect data, graph, and analyze:
  - Favourites: TV show, types of music, musical band, sports team, video games;
  - Numbers: number of pets, brothers/sisters, hours watching TV, hours on Instant; Messaging/computer;
  - Measures: sitting height, arm span, area of foot, time on the bus.
- Present students with "real life" survey questions such as students' satisfaction with cafeteria food, the most popular noon hour activity, or whether students would like to have a school uniform. Have them collect the data, display it with an appropriate graph and interpret the results.
- Give groups of students examples of different types of graphs. Have them create reasons for when and why we would use this type of graph. Combine ideas and have students present their findings. Students could also create a list of questions relating to the graph that could then be analyzed.

SCO: **6.SP2 Graph collected data and analyze the graph to solve problems.**
[C, CN, PS]

Assessment Strategies

- Have students determine an appropriate type of graph for displaying a set of data and justify the choice of graph.
- Ask students to describe the purpose of each type of graph and give examples of appropriate and inappropriate use for each.
- Provide a graph. Ask students to write everything they can tell from it. Have them graph the same data using a different format and discuss which more clearly depicts the information.
- Have students answer a given question by performing an experiment or collecting data. Students should record the results, graph the data, and draw conclusions based on the graph.
- Provide a collection of data and have students graph the information. Consider the student’s choice of graph format and the presence of title, labels, appropriate scales, and accurate data representation.
- Provide a graph and ask students questions that require careful analysis of the data shown.

SCO: **6.SP3 Demonstrate an understanding of probability by:**
- **identifying all possible outcomes of a probability experiment**
- **differentiating between experimental and theoretical probability**
- **determining the theoretical probability of outcomes in a probability experiment**
- **determining the experimental probability of outcomes in a probability experiment**
- **comparing experimental results with the theoretical probability for an experiment.**

[C, ME, PS, T]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Five	Grade Six	Grade Seven
	6.SP3 Demonstrate an understanding of probability by: • identifying all possible outcomes of a probability experiment • differentiating between experimental and theoretical probability • determining the theoretical probability of outcomes in a probability experiment • determining the experimental probability of outcomes in a probability experiment • comparing experimental results with the theoretical probability for an experiment.	**7.SP3** Express probabilities as ratios, fractions and percents. **7.SP4** Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. list the possible outcomes of a probability experiment, such as:
   a. tossing a coin
   b. rolling a die with a given number of sides
   c. spinning a spinner with a given number of sectors;

B. determine the theoretical probability of an outcome occurring for a given probability experiment;

C. predict the probability of a given outcome occurring for a given probability experiment by using theoretical probability;

D. conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability;

E. explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome; and

F. distinguish between theoretical probability and experimental probability, and explain the differences.

SCO: **6.SP3 Demonstrate an understanding of probability by:**

- **identifying all possible outcomes of a probability experiment**
- **differentiating between experimental and theoretical probability**
- **determining the theoretical probability of outcomes in a probability experiment**
- **determining the experimental probability of outcomes in a probability experiment**
- **comparing experimental results with the theoretical probability for an experiment.**

[C, ME, PS, T]

Elaboration

Probability is a measure of how likely an event is to occur. Probability is about predictions of events over the long term rather than predictions of individual, isolated events. **Theoretical probability** can sometimes be obtained by carefully considering the possible outcomes and using the rules of probability. For example, in flipping a coin, there are only two possible outcomes, so the probability of flipping a head is, in theory, $\frac{1}{2}$.

Often in real-life situations involving probability, it is not possible to determine theoretical probability. We must rely on observation of several **trials** (experiments) and a good estimate, which can often be made through a data collection process. This is called **experimental probability**.

Theoretical probability of an event is the ratio of the number of favourable outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely.

$$\textit{Theoretical probability} = \frac{\textit{Number of favourable outcomes}}{\textit{Total number of possible outcomes}}$$

Experimental probability or relative frequency of an event is the ratio of the number of observed successful occurrences of the event to the total number of **trials**. The greater the number of trials, the closer the experimental probability approaches the theoretical probability. Before conducting experiments, students should predict the probability whenever possible.

$$\textit{Experimental probability} = \frac{\textit{Number of observed successful occurrences}}{\textit{Total number of trials in the experiment}}$$

This specific curriculum outcome is addressed in *Math Makes Sense 6* in the following units:

- Unit 7, Lesson 6, pp. 271-275
- Unit 7, Lesson 7, pp. 276-279
- Unit 7, Technology Lesson, p. 280
- Unit 7, Game, p. 281
- Unit 7, Lesson 8, pp. 282, 283
- Unit 7, Unit Problem, pp. 286, 287

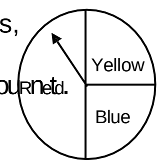
SCO: **6.SP3 Demonstrate an understanding of probability by:**
- **identifying all possible outcomes of a probability experiment**
- **differentiating between experimental and theoretical probability**
- **determining the theoretical probability of outcomes in a probability experiment**
- **determining the experimental probability of outcomes in a probability experiment**
- **comparing experimental results with the theoretical probability for an experiment.**

[C, ME, PS, T]

Instructional Strategies

Consider the following strategies when planning lessons:

- Introduce students to simulations: experiments which indirectly model a situation. Students will have had experience directly determining experimental probabilities in grade 5. An example of a simulation is creating a spinner that represents a basketball player who makes their free throws 8 times in 10. The spinner has 0.8 of the face labelled HIT and 0.2 labelled MISS. This can also be simulated with a 10-sided dice: the numbers 1 to 8 representing HITS and numbers 9 and 10 representing MISSES. Either model can be used to simulate:
  - the probability of making exactly 3 shots in the next 5 tries;
  - the probability of missing the first shot, but making the next 3 in a row;
  - the probability of missing 5 shots in a row.
- Students will often be presented with situations for which outcomes are equally likely. In these cases, they should list the outcomes and count the number of items on the list to determine probabilities. Students must also recognize, however, when outcomes are not equally likely and take this into account. For example, using the spinner shown, the student might list the outcomes as "red," "yellow" and "blue" and assume that since there are 3 outcomes, each has a probability of $\frac{1}{3}$. This, however, is not the case. Students might benefit from reconfiguring the spinner to show equally likely outcomes by dividing the red section into two equal pieces. Now the outcomes might be "red 1," "red 2," "yellow" and "blue" and each outcome would now have a probability of $\frac{1}{4}$. Because there are two red sections, the probability of red is, therefore, $\frac{2}{4}$.



Suggested Activities

- Have students determine approximately how many boxes of cereal will need to be purchased before a consumer collects each of six possible prizes contained therein. This simulation can be performed by rolling a die, recording the prize number won (based on the roll of the die), continuing until at least one of each number is rolled, repeating the experiment several times and determining, on average, the number of rolls (purchases) required.
- Tell students that a particular baseball player has an average of .250 (i.e., he gets 1 hit in 4 times at bat, on average). Ask students to conduct a simulation to determine the probability that the player will get a hit each time at bat in a particular game.
- Have students explain how a scientific experiment is like a probability experiment, explaining the differences between theory/hypothesis and experimental results?

SCO: **6.SP3 Demonstrate an understanding of probability by:**

- **identifying all possible outcomes of a probability experiment**
- **differentiating between experimental and theoretical probability**
- **determining the theoretical probability of outcomes in a probability experiment**
- **determining the experimental probability of outcomes in a probability experiment**
- **comparing experimental results with the theoretical probability for an experiment.**

[C, ME, PS, T]

Assessment Strategies

- Ask students to create a spinner for which there are 6 equally likely outcomes and another spinner for which the 6 outcomes are not equally likely. Have them predict the probability for each spinner's outcomes.
- Ask students: Why would you use a die to conduct a simulation to determine the number of cereal boxes you would need to purchase to collect each of six possible prizes, but a different device if there were 10 possible prizes?
- Have students predict how many goals will be scored in 24 shots by rolling a die where the number 1 is a goal and the numbers 2 to 6 are misses. Conduct the experiment, and compare results for more than one team.
- Ask students to list the equally likely outcomes that result when two dice are rolled and the numbers are subtracted, conduct the experiment 6 times and compare theoretical and experimental. Then conduct 12 times and compare with the first results. Explain what happens when you increase the number of trials in a probability experiment.
- Ask students to list the equally likely outcomes that result when two cubes are pulled from a bag with 10 red cubes and 5 blue ones.
- Have students explain how a scientific experiment is like a probability experiment, explaining the differences between theory/hypothesis and experimental results?

REFERENCES

Alberta Education. *LearnAlberta.ca: Planning Guides K, 1, 4, and 7*, 2005-2008.

American Association for the Advancement of Science [AAAS-Benchmarks]. *Benchmark for Science Literacy*. New York, NY: Oxford University Press, 1993.

Banks, J.A. and C.A.M. Banks. *Multicultural Education: Issues and Perspectives*. Boston: Allyn and Bacon, 1993.

Black, Paul and Dylan Wiliam. “Inside the Black Box: Raising Standards Through Classroom Assessment.” *Phi Delta Kappan*, 20, October 1998, pp.139-148.

British Columbia. Ministry of Education. *The Primary Program: A Framework for Teaching*, 2000.

Caine, Renate Numella and Geoffrey Caine. *Making Connections: Teaching and the Human Brain*. Menlo Park, CA: Addison-Wesley Publishing Company, 1991.

Computation, Calculators, and Common Sense. May 2005, NCTM.

Davies, Anne. *Making Classroom Assessment Work*. British Columbia: Classroom Connections International, Inc., 2000.

Hope, Jack A. et.al. *Mental Math in the Primary Grades* (p. v). Dale Seymour Publications, 1988.

National Council of Teachers of Mathematics (NCTM). *Curriculum Focal Points for Prekindergarten through Grade 8: A Quest for Coherence*. Reston, VA: NCTM, 2006.

_____, *Mathematics Assessment Sampler, Grades 3-5*. edited by Jane Reston, VA: NCTM, 2000.

_____, *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

OECD Centre for Educational Research and Innovation. *Formative Assessment: Improving Learning in Secondary Classrooms*. Paris, France: Organization for Economic Co-operation and Development (OECD) Publishing, 2006.

Rubenstein, Rheta N. *Mental Mathematics beyond the Middle School: Why? What? How?* September 2001, Vol. 94, Issue 6, p. 442.

Shaw, J.M. and Cliatt, M.F.P. (1989). “Developing Measurement Sense.” In P.R. Trafton (Ed.), *New Directions for Elementary School Mathematics* (pp. 149–155). Reston, VA: National Council of Teachers of Mathematics.

Small, M. *Making Math Meaningful to Canadian Students, K-8*. Toronto: Nelson Education Ltd., 2008.

Steen, L.A. (ed.). *On the Shoulders of Giants – New Approaches to Numeracy.* Washington, DC: National Research Council, 1990.

Stenmark, Jean Kerr and William S. Bush, Editor. *Mathematics Assessment: A Practical Handbook for Grades 3-5*. Reston, VA: National Council of Teachers of Mathematics, Inc., 2001.

Trilling Bernie and Charles Fadel, *21st Century Skills, Learning for Life in our Times*. San Francisco: John Wiley & Sons, Inc. 2009

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades K-3.* Boston: Pearson Education, Inc. 2006.

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades 3-5.* Boston: Pearson Education, Inc. 2006

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades 5-8.* Boston: Pearson Education, Inc. 2006.

Western and Northern Canadian Protocol. *Common Curriculum Framework for K-9 Mathematics*, 2006.