Prince Edward Island
Canada

Education, Early Learning and Culture
English Programs

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

MAT421K

Revised, 2015
Prince Edward Island
Department of Education,
Early Learning and Culture
Holman Centre
250 Water Street, Suite 101
Summerside, Prince Edward Island
Canada, C1N 1B6
Tel: (902) 438-4130
Fax: (902) 438-4062
www.gov.pe.ca/eecd/

Acknowledgments

The Department of Education and Early Childhood Development of Prince Edward Island gratefully acknowledges the contributions of the following groups and individuals toward the development of the *Prince Edward Island MAT421K Mathematics Curriculum Guide*:

- The following specialist from the Prince Edward Island Department of Education and Early Childhood Development:

  J. Blaine Bernard,
  Secondary Mathematics Specialist,
  Department of Education and
  Early Childhood Development

- The 2010-2011 MAT421K pilot teachers:

  Debbie Caissy,
  Kinkora Regional High School

  Cecil Keeping,
  Souris Regional High School

  Randy Nicholson,
  Bluefield High School

- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education

- Alberta Education

Table of Contents

Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for Grades 10-12 Mathematics* (2008) has been adopted as the basis for a revised high school mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential Graduation Learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

➢ Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for 10-12 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

TOPICS \ GRADE	10 11 12
The topics vary in the courses from grades ten to twelve. These topics include: **Algebra** **Calculus** **Financial Mathematics** **Geometry** **Logical Reasoning** **Mathematics Research Project** **Measurement** **Number** **Permutations, Combinations, and the Binomial Theorem** **Probability** **Relations and Functions** **Statistics** **Trigonometry**	**GENERAL CURRICULUM OUTCOMES (GCOs)** **SPECIFIC CURRICULUM OUTCOMES (SCOs)** **ACHIEVEMENT INDICATORS**
MATHEMATICAL PROCESSES	**Communication, Connections, Reasoning, Mental Mathematics and Estimation, Problem Solving, Technology, Visualization**

NATURE OF MATHEMATICS
Change **Constancy** **Number Sense** **Patterns** **Relationships** **Spatial Sense** **Uncertainty**

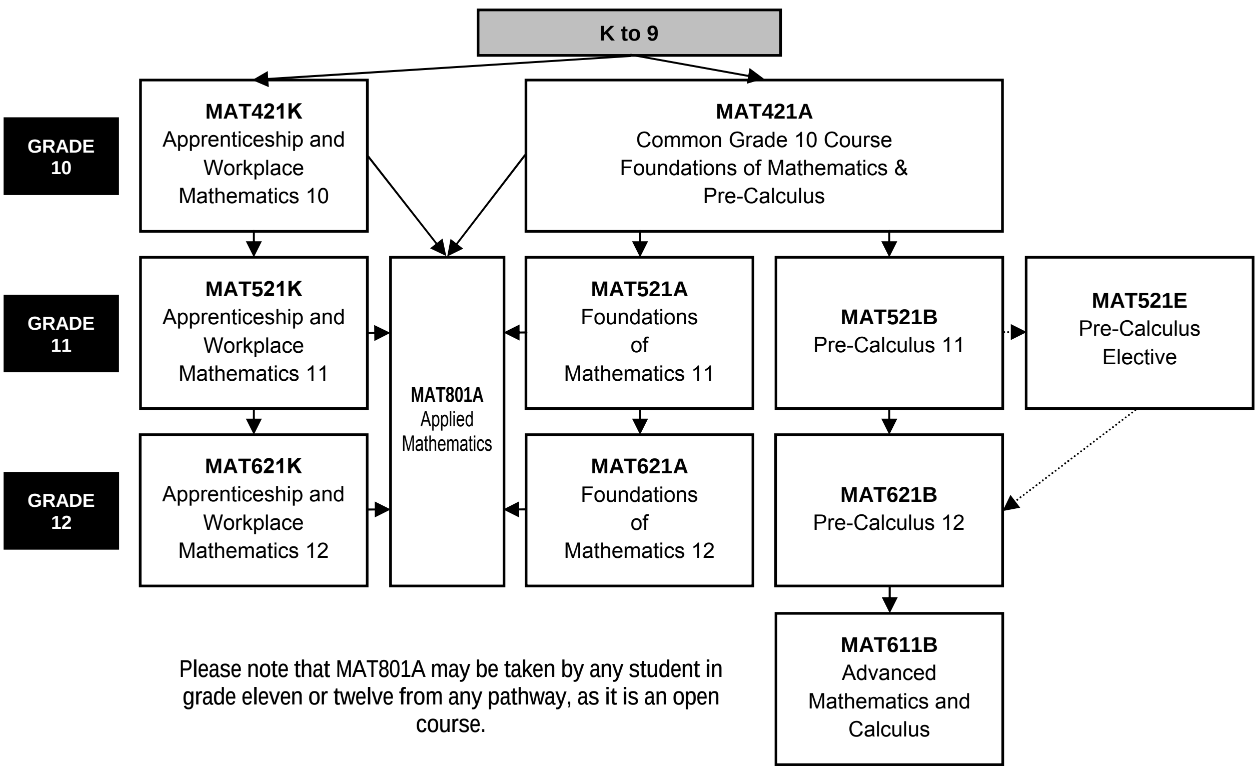
The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into a number of topics, as described above. These topics are not intended to be discrete units of instruction. The integration of outcomes across topics makes mathematical experiences meaningful. Students should make the connections among concepts both within and across topics. Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.

- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Pathways and Topics

The Prince Edward Island 10-12 mathematics curriculum includes pathways with corresponding topics rather than strands, which are found in the Prince Edward Island K-9 mathematics curriculum. Three pathways are available: **Apprenticeship and Workplace Mathematics**, **Foundations of Mathematics**, and **Pre-Calculus**. A common grade ten course (Foundations of Mathematics and Pre-Calculus, Grade 10) is the starting point for the Foundations of Mathematics pathway and the Pre-Calculus pathway. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings. These pathways are illustrated in the diagram below:



The goals of all three pathways are to provide the prerequisite knowledge, skills, understandings, and attitudes for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study or for direct entry into the work force. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Apprenticeship and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics, and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, algebra and number, logical reasoning, relations and functions, statistics, probability, and a mathematics research project.

Pre-Calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, combinatorics, and introductory calculus.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

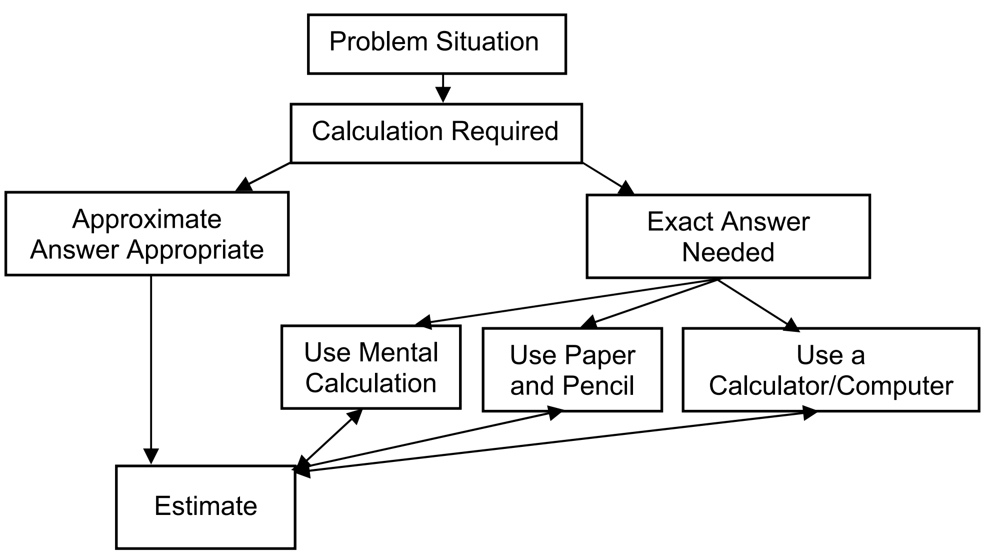
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001). Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?" or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;

- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.

- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all topics in mathematics and it is important that connections are made among topics. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical topics. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education" (p.60). The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate “communicating to learn mathematics and learning to communicate mathematically” (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

➢ Inquiry-Based Learning and Project Based Learning

Inquiry-based learning (IBL) allows students to explore, investigate, and construct new meaning from prior knowledge and from new information that is retrieved from other sources. It is not linear in nature, but promotes a continual looping back and forth throughout the process as students gather and process new information, redirect their inquiries, and continue through the process. Mathematical inquiry will require students to practise and refine their critical and creative-thinking skills. The terms *inquiry* and *research* are often used interchangeably within an educational context. While research often becomes the end-result of an inquiry process, it is the process itself that should be emphasized within an educational context. More information regarding the development of a mathematics research project is included in the appendix at the end of this document.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, teaching has been effective, or how best to address student learning needs. The quality of assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

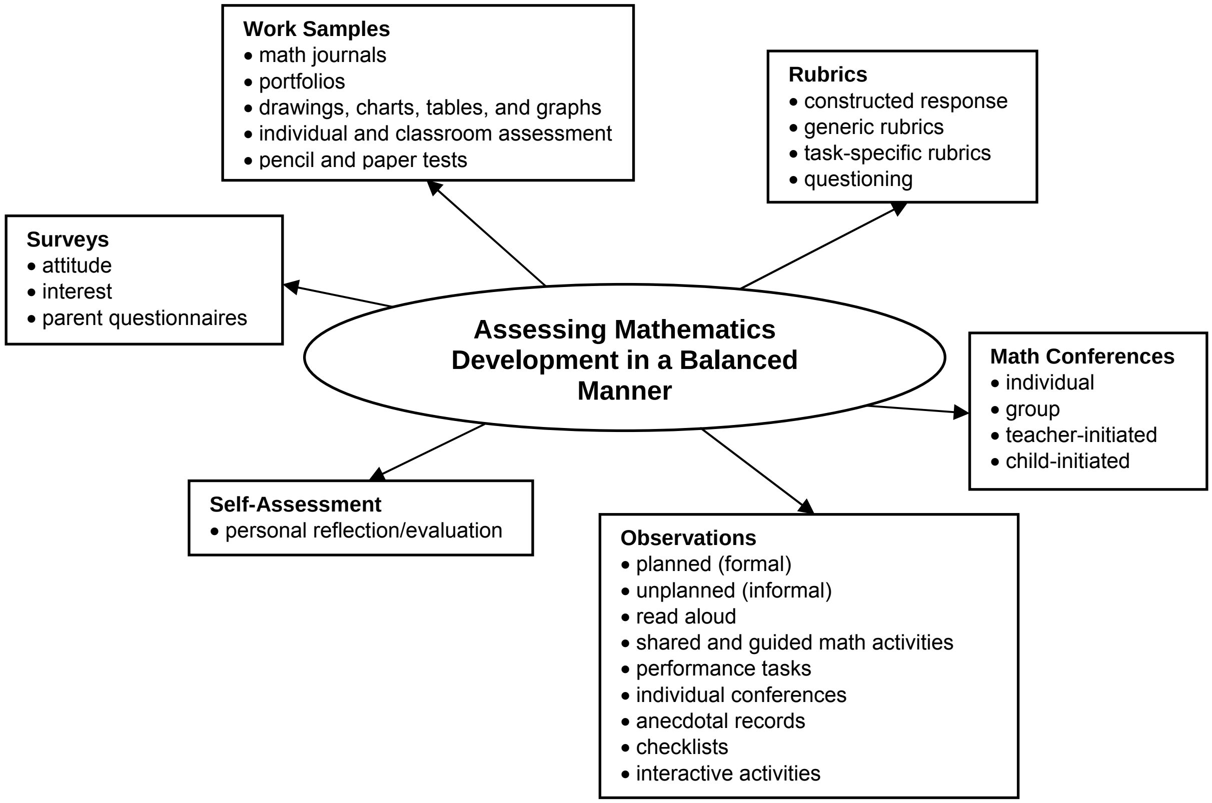
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;

- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, phone calls, and electronic methods.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;

- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he or she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island high school mathematics curriculum are organized into a number of topics across the grades from ten to twelve. Each topic has associated with it a general curriculum outcome (GCO). They are overarching statements about what students are expected to learn in each topic from grades ten to twelve.

Topic	General Curriculum Outcome (GCO)
Algebra (A)	Develop algebraic reasoning.
Algebra and Number (AN)	Develop algebraic reasoning and number sense.
Calculus (C)	Develop introductory calculus reasoning.
Financial Mathematics (FM)	Develop number sense in financial applications.
Geometry (G)	Develop spatial sense.
Logical Reasoning (LR)	Develop logical reasoning.
Mathematics Research Project (MRP)	Develop an appreciation of the role of mathematics in society.
Measurement (M)	Develop spatial sense and proportional reasoning. (***Foundations of Mathematics and Pre-Calculus***)
	Develop spatial sense through direct and indirect measurement. (***Apprenticeship and Workplace Mathematics***)
Number (N)	Develop number sense and critical thinking skills.
Permutations, Combinations and Binomial Theorem (PC)	Develop algebraic and numeric reasoning that involves combinatorics.
Probability (P)	Develop critical thinking skills related to uncertainty.
Relations and Functions (RF)	Develop algebraic and graphical reasoning through the study of relations.
Statistics (S)	Develop statistical reasoning.
Trigonometry (T)	Develop trigonometric reasoning.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific curriculum outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding topic and general curriculum outcome;
- the scope and sequence of the specific curriculum outcome(s) from grades ten to eleven which correspond to this SCO;
- the specific curriculum outcome, with a list of achievement indicators;

- a list of the sections in *Math at Work 10* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

In the second half of this document, a curriculum guide supplement is presented which follows the primary resource, *Math at Work 10*. As well, an appendix is included which outlines the steps to follow in the development of an effective mathematics research project.

MEASUREMENT

SPECIFIC CURRICULUM OUTCOMES

M1 – Demonstrate an understanding of the Système International (SI) by:

- **describing the relationships of the units for length, area, volume, capacity, mass and temperature;**
- **applying strategies to convert SI units to imperial units.**

M2 – Demonstrate an understanding of the imperial system by:

- **describing the relationships of the units for length, area, volume, capacity, mass and temperature;**
- **comparing the American and British imperial units for capacity;**
- **applying strategies to convert imperial units to SI units.**

M3 – Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.

M4 – Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.

MAT421K – Topic: Measurement (M)

GCO: Develop spatial sense through direct and indirect measurement.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
M1 Demonstrate an understanding of the Système International (SI) by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature; • applying strategies to convert SI units to imperial units.	

SCO: **M1 – Demonstrate an understanding of the Système International (SI) by:**

- **describing the relationships of the units for length, area, volume, capacity, mass and temperature;**
- **applying strategies to convert SI units to imperial units.**

[C, CN, ME, V]

Students who have achieved this outcome should be able to:

A. Explain how the SI system was developed, and explain its relationship to base ten.

B. Identify the base units of measurement in the SI system, and determine the relationship among the related units of each type of measurement.

C. Identify contexts that involve the SI system.

D. Match the prefixes used for SI units of measurements with the powers of ten.

E. Explain, using examples, how and why decimals are used in the SI system.

F. Provide an approximate measurement in SI units for a measurement given in imperial units; *e.g.,* 1 inch is approximately 2.5 cm.

G. Write a given linear measurement expressed in one SI unit in another SI unit.

H. Convert a given measurement from SI to imperial units by using proportional reasoning (including formulas); *e.g.,* Celsius to Fahrenheit, centimetres to inches.

Note: It is intended that this outcome be limited to the base units and the prefixes *milli-*, *centi-*, *deci-*, *deca-*, *hecto-*, and *kilo-*.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.3 (A B C D E F H)

2.2 (B C G)

2.3 (F H)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: M1 – Demonstrate an understanding of the Système International (SI) by:
- **describing the relationships of the units for length, area, volume, capacity, mass and temperature;**
- **applying strategies to convert SI units to imperial units.**

[C, CN, ME, V]

Elaboration

The International System of Units (abbreviated SI from the French *le Système international d'unités*) is the modern form of the metric system which is generally a system of units of measurement devised around a number of base units and the convenience of the number ten. It is the world's most widely used system of measurement, both in everyday commerce and in science. The original metric system was developed in France in the late 1700s. In Canada, we use the SI system and the imperial system. Although we use the SI most often in our daily lives, the imperial system is used in many trades.

In the SI system, the base unit for measuring length is the metre (m). The base unit for measuring volume and capacity is the litre (L), and the base unit for measuring mass is the kilogram (kg). In the SI system, temperatures are measured using the Celsius scale. Because the Celsius system is a 100-step scale from the freezing to the boiling point of water, it is sometimes referred to as a centigrade scale, from the Latin words meaning *hundred steps*. Temperatures can be converted from SI units to imperial units by using the formula $F = \frac{9}{5}C + 32$, where C is measured in degrees Celsius and F is measured in degrees Fahrenheit.

The SI system is a decimal system because it is based on powers of 10. Any measurement stated in one SI unit can be converted to another SI unit by multiplying or dividing by a power of 10. Powers of the base units are indicated by SI prefixes, as is shown in the table below.

PREFIX	ABBREVIATION	MULTIPLYING FACTOR
kilo-	k	1000
hecto-	h	100
deca-	da	10
deci-	d	0.1
centi-	c	0.01
milli-	m	0.001

The following are some common conversions between SI and imperial units:

Exact Conversions

1 in = 2.54 cm	1 ft = 30.48 cm	1 yd = 0.9144 m

Approximate Conversions

Rounded to four significant digits

1 mm ≐ 0.03937 in	1 cm ≐ 0.3937 in	1 m ≐ 1.094 yd
1 m ≐ 3.281 ft	1 km ≐ 0.6214 mi	1 mi ≐ 1.609 km
1 kg ≐ 2.205 lb	1 lb ≐ 0.4536 lb	

MAT421K – Topic: Measurement (M)

GCO: Develop spatial sense through direct and indirect measurement.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
M2 Demonstrate an understanding of the imperial system by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature; • comparing the American and British imperial units for capacity; • applying strategies to convert imperial units to SI units.	

SCO: **M2 – Demonstrate an understanding of the imperial system by:**

- **describing the relationships of the units for length, area, volume, capacity, mass and temperature;**
- **comparing the American and British imperial units for capacity;**
- **applying strategies to convert imperial units to SI units.**

[C, CN, ME, V]

Students who have achieved this outcome should be able to:

A. Explain how the imperial system was developed.

B. Identify commonly used units in the imperial system, and determine the relationships among the related units.

C. Identify contexts that involve the imperial system.

D. Explain, using examples, how and why fractions are used in the imperial system.

E. Compare the American and British imperial measurement systems; *e.g.,* gallons, bushels, tons.

F. Provide an approximate measurement in imperial units for a measurement given in SI units; *e.g.,* 1 litre is approximately $\frac{1}{4}$ US gallon.

G. Write a given linear measurement expressed in one imperial unit in another imperial unit.

H. Convert a given measurement from imperial to SI units by using proportional reasoning (including formulas); *e.g.,* Fahrenheit to Celsius, inches to centimetres.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.3 (A B C D E F H)

2.1 (B C G)

2.3 (F H)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: M2 – Demonstrate an understanding of the imperial system by:

- **describing the relationships of the units for length, area, volume, capacity, mass and temperature;**
- **comparing the American and British imperial units for capacity;**
- **applying strategies to convert imperial units to SI units.**

[C, CN, ME, V]

Elaboration

The imperial system is a collection of units that were developed at different times to meet different needs. As a result, each group of units has a particular relationship. Because of this fact, the imperial system is not a decimal system, as is the SI system.

This system can be traced back to the Roman Empire. It has been modified over time but is essentially based on unit sizes which have proved to be useful or convenient. Maybe the most interesting are the measurements of capacity. In general, these are based on a square based box. Double the size of the base and the volume increases by four. Double the height and the increase is only two. A similar system applies to measurements below one inch. Because of these comparisons, smaller sizes are usually measured in fractions of a unit.

In the imperial system, some of the units of measurement are inches and feet for measuring length, pints and quarts for measuring capacity, and pounds and ounces for measuring weight. In the imperial system, temperatures are measured using the Fahrenheit scale. Temperatures can be converted from imperial units to SI units by using the formula $C = \frac{5}{9}(F - 32)$, where F is measured in degrees Fahrenheit and C is measured in degrees Celsius.

The imperial system of measurement is widely used in the United States for measuring distances. Even though SI is Canada's official measurement system, some Canadian industries still use imperial units. The following units are the basic imperial units used for measuring distances.

inch (in)	
foot (ft)	1 ft = 12 in
yard (yd)	1 yd = 3 ft = 36 in
mile (mi)	1 mi = 1760 yd = 5280 ft

Even though both the imperial system and the American system of measurement are based on the older English units of measurement, there are some differences between them. After the United States became independent, they developed their own measurement standards, based on the English units system which was used throughout the States prior to independence. For example, an American gallon is slightly less than an imperial gallon.

MAT421K – Topic: Measurement (M)

GCO: Develop spatial sense through direct and indirect measurement.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
M3 Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.	

SCO: **M3 – Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.** [CN, ME, PS, V]

Students who have achieved this outcome should be able to:

A. Identify a referent for a given common SI or imperial unit of linear measurement.

B. Estimate a linear measurement, using a referent.

C. Measure inside diameters, outside diameters, lengths, widths of various given objects, and distances, using various measuring instruments.

D. Estimate the dimensions of a given regular 3-D object or 2-D shape, using a referent; *e.g.,* the height of the desk is about three rulers long, so the desk is approximately 3 ft high.

E. Solve a linear measurement problem including perimeter, circumference, and $\text{length} + \text{width} + \text{height}$ (used in shipping and air travel).

F. Determine the operation that should be used to solve a linear measurement problem.

G. Provide an example of a situation in which a fractional linear measurement would be divided by a fraction.

H. Determine, using a variety of strategies, the midpoint of a linear measurement such as length, width, height, depth, diagonal and diameter of a 3-D object, and explain the strategies.

I. Determine if a solution to a problem that involves linear measurement is reasonable.

Note: It is intended that the four arithmetic operations on decimals and fractions be integrated into the problems.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B C D E)

2.2 (A B C D E)

2.4 (C D E F G H I)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: M3 – Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements. [CN, ME, PS, V]

Elaboration

To become comfortable with various SI and imperial units of linear measurement, students should be encouraged to develop referents for them. For example, the thickness of a dime is approximately one millimetre, a thumb length is approximately one inch, and the distance walked in 20 minutes is approximately one mile. These, and other referents can be used to estimate linear measurements.

Students should be familiar with a variety of measuring instruments such as rulers, measuring tapes and calipers to help them measure various dimensions of objects, such as diameters, lengths and widths.

Students should also be familiar with some common formulas for perimeter and circumference such as:

Perimeter of a Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference of a Circle	$C = 2\pi r$ or $C = \pi d$

MAT421K – Topic: Measurement (M)

GCO: Develop spatial sense through direct and indirect measurement.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
M4 Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.	**M1** Solve problems that involve SI and imperial units in surface area measurements and verify the solutions. **M2** Solve problems that involve SI and imperial units in volume and capacity measurements. **G3** Model and draw 3-D objects and their views. **G4** Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.

SCO: **M4 – Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.** [ME, PS, R, V]

Students who have achieved this outcome should be able to:

A. Identify and compare referents for area measurements in SI and imperial units.

B. Estimate an area measurement, using a referent.

C. Identify a situation where a given SI or imperial area unit would be used.

D. Estimate the area of a given regular, composite or irregular 2-D shape, using an SI square grid and an imperial square grid.

E. Solve a contextual problem that involves the area of a regular, a composite or an irregular 2-D shape.

F. Write a given area measurement expressed in one SI unit squared in another SI unit squared.

G. Write a given area measurement expressed in one imperial unit squared in another imperial unit squared.

H. Solve a problem, using formulas for determining the areas of regular, composite and irregular 2-D shapes, including circles.

I. Solve a problem that involves determining the surface area of 3-D objects, including right cylinders and cones.

J. Explain, using examples, the effect of changing the measurement of one or more dimensions on area and perimeter of rectangles.

K. Determine if a solution to a problem that involves an area measurement is reasonable.

Note: It is intended that the four arithmetic operations on decimals and fractions be integrated into the problems.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.1 (A B C D E G H K)

3.2 (A B C D E F H K)

3.3 (D E J)

3.4 (I)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: M4 – Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions. [ME, PS, R, V]

Elaboration

Students should be familiar with some common formulas for area and surface area such as:

Area of a Rectangle	$A = lw$
Area of a Triangle	$A = \dfrac{bh}{2}$
Area of a Circle	$A = \pi r^2$
Surface Area of a Rectangular Prism	$SA = 2(lw + lh + wh)$
Surface Area of a Cylinder	$SA = 2\pi r^2 + 2\pi rh$

When converting from one unit squared to another unit squared, remind students that they have to apply the conversion factor twice. For example, to covert from m^2 to cm^2, we use the following procedure:

$$6.24\ \text{m}^2 = 6.24 \times (100\ \text{cm})^2, \qquad \text{since } 1\ \text{m} = 100\ \text{cm}$$

$$6.24\ \text{m}^2 = 6.24 \times (100\ \text{cm}) \times (100\ \text{cm})$$

$$6.24\ \text{m}^2 = 62{,}400\ \text{cm}^2$$

GEOMETRY

SPECIFIC CURRICULUM OUTCOMES

G1 – Analyse puzzles and games that involve spatial reasoning, using problem-solving strategies.

G2 – Demonstrate an understanding of the Pythagorean theorem by:
- **identifying situations that involve right triangles;**
- **verifying the formula;**
- **applying the formula;**
- **solving problems.**

G3 – Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.

G4 – Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by:
- **applying similarity to right triangles;**
- **generalizing patterns from similar right triangles;**
- **applying the primary trigonometric ratios;**
- **solving problems.**

G5 – Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them.

G6 – Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by:
- **drawing;**
- **replicating and constructing;**
- **bisecting;**
- **solving problems.**

MAT421K – Topic: Geometry

(G) GCO: Develop spatial sense.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
G1 Analyse puzzles and games that involve spatial reasoning, using problem-solving strategies.	

SCO: **G1 – Analyse puzzles and games that involve spatial reasoning, using problem-solving strategies.**
[C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Determine, explain and verify a strategy to solve a puzzle or to win a game; *e.g.,*
- guess and check;
- look for a pattern;
- make a systematic list;
- draw or model;
- eliminate possibilities;
- simplify the original problem;
- work backward;
- develop alternative approaches.

B. Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

C. Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Note: It is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction and similar puzzles and games.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

Integrated throughout the text.

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: G1 – Analyse puzzles and games that involve spatial reasoning, using problem-solving strategies.
[C, CN, PS, R]

Elaboration

This particular outcome is integrated throughout the course. Each chapter has a section called *Games and Puzzles* that can be used to meet this particular SCO. They are found on pages 53, 111, 169, 217, 279, 319, and 381 of the textbook.

MAT421K – Topic: Geometry

(G) GCO: Develop spatial sense.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
G2 Demonstrate an understanding of the Pythagorean theorem by: • identifying situations that involve right triangles; • verifying the formula; • applying the formula; • solving problems.	**G1** Solve problems that involve two and three right triangles.

SCO: **G2 – Demonstrate an understanding of the Pythagorean theorem by:**

- **identifying situations that involve right triangles;**
- **verifying the formula;**
- **applying the formula;**
- **solving problems.**

[C, CN, PS, V]

Students who have achieved this outcome should be able to:

A. Explain, using illustrations, why the Pythagorean theorem only applies to right triangles.

B. Verify the Pythagorean theorem, using examples and counterexamples, including drawings, concrete materials and technology.

C. Describe historical and contemporary applications of the Pythagorean theorem.

D. Determine if a given triangle is a right triangle, using the Pythagorean theorem.

E. Explain why a triangle with the side length ratio of $3:4:5$ is a right triangle.

F. Explain how the ratio of $3:4:5$ can be used to determine if a corner of a given 3-D object is square (90^0) or if a given parallelogram is a rectangle.

G. Solve a problem, using the Pythagorean theorem.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (A)

6.2 (B C G)

6.3 (D E F G)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: G2 – Demonstrate an understanding of the Pythagorean theorem by:

- **identifying situations that involve right triangles;**
- **verifying the formula;**
- **applying the formula;**
- **solving problems.**

[C, CN, PS, V]

Elaboration

Pythagoras of Samos, *c*. 560 BC – *c*. 480 BC, was a Greek philosopher who is credited with providing the first proof of the Pythagorean relationship. It states that the area of the square on the hypotenuse of a right triangle is equal to the sum of the areas of the squares on the other two sides of the triangle. The conventional formula for the Pythagorean relationship, $c^2 = a^2 + b^2$, should be developed through investigations. It is also important for students to recognize that the Pythagorean relationship can be labelled differently from the conventional *a-b-c* notation. Using this notation, the hypotenuse, or the longest side, is *c* and two shorter sides, or legs, are *a* and *b*.

A Pythagorean triple is any set of three whole numbers *a, b* and *c*, for which $a^2 + b^2 = c^2$. It is believed that the Egyptians and other ancient cultures used a 3-4-5 rule $(a = 3,\ b = 4,\ c = 5)$ in construction to ensure buildings were square. The 3-4-5 rule allowed them a quick method of establishing a right angle. This method is still used today in construction.

In presenting diagrams of right triangles, it is important to give diagrams of the triangles in various orientations. Students should recognize the hypotenuse as being the side opposite the right angle, regardless of the orientation of the figure. Whenever a triangle has a right angle and two known side lengths, the Pythagorean relationship should be recognized by students. In addition to being provided with situations that involve finding the length of the hypotenuse, students should also be given situations where the hypotenuse and one side is known and the other side is to be found. Also, it is important for students to realize that they can use the Pythagorean relationship when only one side is known if the right triangle is isosceles. Finally, students should be able to use the Pythagorean relationship to determine if three given side lengths are, or are not, the sides of a right triangle. There are many opportunities to use the Pythagorean relationship to solve problems, such as determining the height of a building or finding the shortest distance across a rectangular field.

Students need to be provided with opportunities to model and explain the Pythagorean theorem concretely, pictorially, and symbolically:

- **Concretely** – by cutting up areas represented by a^2 and b^2, and fitting the two areas onto c^2
- **Pictorially** – using grid paper or technology
- **Symbolically** – by confirming that a right triangle is formed by showing that $a^2 + b^2 = c^2$

MAT421K – Topic: Geometry

(G) GCO: Develop spatial sense.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
G3 Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.	**G2** Solve problems that involve scale.

SCO: **G3 – Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.** [C, CN, PS, V]

Students who have achieved this outcome should be able to:

A. Determine, using angle measurements, if two or more regular or irregular polygons are similar.

B. Determine, using ratios of side lengths, if two or more regular or irregular polygons are similar.

C. Explain why two given polygons are not similar.

D. Explain the relationships between the corresponding sides of two polygons that have corresponding angles of equal measure.

E. Draw a polygon that is similar to a given polygon.

F. Explain why two or more right triangles with a shared acute angle are similar.

G. Solve a contextual problem that involves similarity of polygons.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.1 (A B C D E F G)

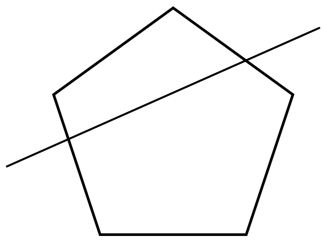
[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

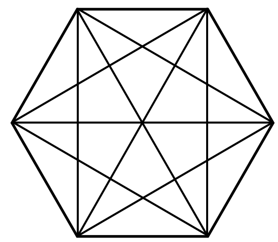
SCO: G3 – Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons. [C, CN, PS, V]

Elaboration

A convex polygon is defined as a polygon with all its interior angles less than 180^0. This means that all the vertices of the polygon will point outwards, away from the interior of the shape. Because of this property, a triangle is always convex.

A line drawn through a convex polygon will intersect the polygon exactly twice, as can be seen from the figure below left. You can also see that the line will divide the polygon into exactly two pieces.



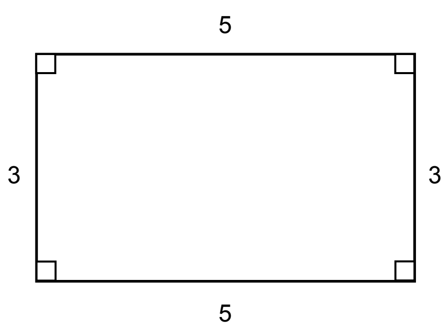


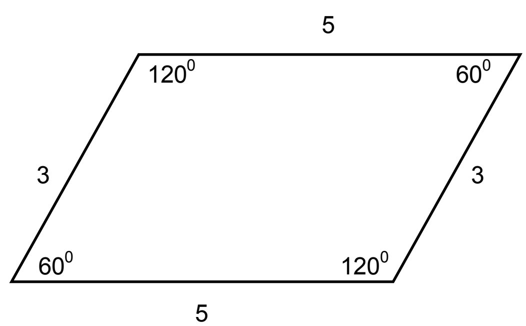
All the diagonals of a convex polygon lie entirely inside the polygon, as is shown in the figure above right. In a concave polygon, some diagonals will lie outside the polygon. The area of an irregular convex polygon can be found by dividing it into triangles and adding the areas of the triangles.

Two polygons with the same shape are called similar polygons**.** The symbol for "is similar to" is ~. When two polygons are similar, these two facts must both be true:

- Corresponding angles are equal.
- The ratios of pairs of corresponding sides must all be equal.

If two polygons only have one of these two facts, then they are not similar, as can easily be seen in the following diagram. Even though the ratios of the corresponding sides are equal, the corresponding angles are not equal.





MAT421K – Topic: Geometry

(G) GCO: Develop spatial sense.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
G4 Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: • applying similarity to right triangles; • generalizing patterns from similar right triangles; • applying the primary trigonometric ratios; • solving problems.	**G1** Solve problems that involve two and three right triangles.

SCO: **G4 – Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by:**

- **applying similarity to right triangles;**
- **generalizing patterns from similar right triangles;**
- **applying the primary trigonometric ratios;**
- **solving problems.**

[CN, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Show, for a specified angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the side adjacent are equal, and generalize a formula for the tangent ratio.

B. Show, for a specified angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the hypotenuse are equal, and generalize a formula for the sine ratio.

C. Show, for a specified angle in a set of similar right triangles, that the ratios of the length of the side adjacent to the length of the hypotenuse are equal, and generalize a formula for the cosine ratio.

D. Identify situations where the trigonometric ratios are used for indirect measurement of angles and lengths.

E. Solve a contextual problem that involves right triangles, using the primary trigonometric ratios.

F. Determine if a solution to a problem that involves primary trigonometric ratios is reasonable.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.2 (A D E F)

7.3 (B C D E F)

7.4 (E F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

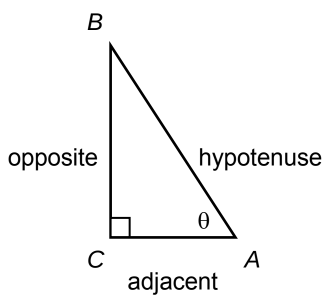
SCO: G4 – Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by:

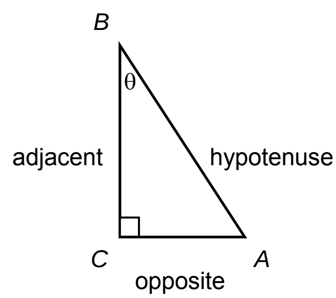
- **applying similarity to right triangles;**
- **generalizing patterns from similar right triangles;**
- **applying the primary trigonometric ratios;**
- **solving problems.**

[CN, PS, R, T, V]

Elaboration

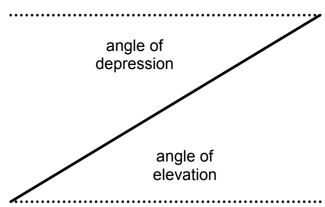
In similar triangles, corresponding angles are equal, and corresponding sides are in proportion. Therefore, the ratios of the lengths of corresponding sides are equal. The sides of a right triangle are labelled according to a reference angle, θ.





A trigonometric ratio is a ratio of the measures of two sides of a right triangle. The three primary trigonometric ratios are tangent, sine and cosine. The short form for the tangent ratio of angle *A* is tan *A*. It is defined as $\tan A = \dfrac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$. The short form for the sine ratio of angle *A* is sin *A*. It is defined as $\sin A = \dfrac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$. The short form for the cosine ratio of angle *A* is cos *A*. It is defined as $\cos A = \dfrac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$.

The line of sight is an invisible line from one person or object to another person or object. Some applications of trigonometry involve an angle of elevation or an angle of depression. An *angle of elevation* is the angle formed by the horizontal and a line of sight above the horizontal. An *angle of depression* is the angle formed by the horizontal and a line of sight below the horizontal. As the diagram below shows, the angle of elevation and the angle of depression along the same line of sight are equal.



MAT421K – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
G5 Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them.	

SCO: **G5 – Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them.** [C, CN, PS, V]

Students who have achieved this outcome should be able to:

A. Sort a set of lines as perpendicular, parallel or neither, and justify this sorting.

B. Illustrate and describe complementary and supplementary angles.

C. Identify, in a set of angles, adjacent angles that are not complementary or supplementary.

D. Identify and name pairs of angles formed by parallel lines and a transversal, including corresponding angles, vertically opposite angles, alternate interior angles, alternate exterior angles, interior angles on the same side of a transversal, and exterior angles on the same side of a transversal.

E. Explain and illustrate the relationships of angles formed by parallel lines and a transversal.

F. Explain, using examples, why the angle relationships do not apply when the lines are not parallel.

G. Determine if lines or planes are perpendicular or parallel, *e.g.,* wall perpendicular to floor, and describe the strategy used.

H. Determine the measures of angles involving parallel lines and a transversal, using angle relationships.

I. Solve a contextual problem that involves angles formed by parallel lines and a transversal (including perpendicular transversals).

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.3 (A B C D E F G)

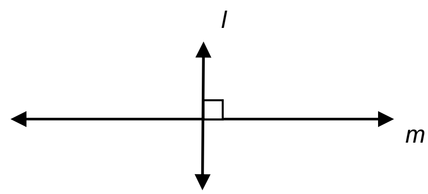
5.4 (H I)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

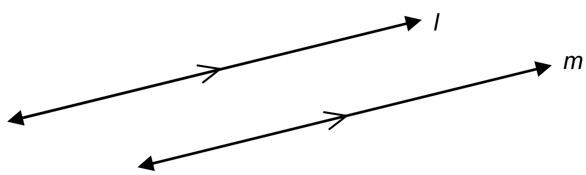
SCO: G5 – Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them. [C, CN, PS, V]

Elaboration

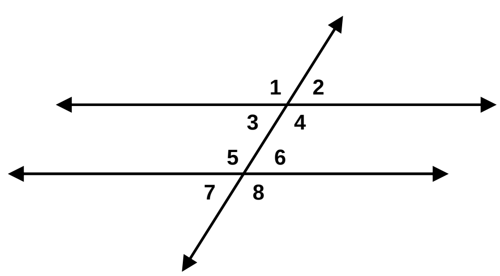
Two lines that intersect and form right angles are called perpendicular lines. The symbol ⊥ is used to denote perpendicular lines. In the figure below, *l* ⊥ *m*.



Two lines, both in the same plane, that never intersect are called parallel lines. Parallel lines remain the same distance apart at all times. The symbol || is used to denote parallel lines. In the figure below , *l* || *m*.



When parallel lines get crossed by another line, which is called a transversal, we can see that many of the angles that are formed are equal, as is shown in the diagram below. In this diagram, angles 1, 4, 5 and 8 are equal; and angles 2, 3, 6 and 7 are equal. These angles can be made into pairs of angles which have special names. In particular, angles 1 and 4 are called *vertical angles*, and angles 1 and 5 are called *corresponding angles*.



MAT421K – Topic: Geometry

(G) GCO: Develop spatial sense.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
G6 Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: • drawing; • replicating and constructing; • bisecting; • solving problems.	

SCO: G6 – Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by:

- **drawing;**
- **replicating and constructing;**
- **bisecting;**
- **solving problems.**

[C, ME, PS, T, V]

Students who have achieved this outcome should be able to:

A. Draw and describe angles with various measures, including acute, right, straight, obtuse, and reflex angles.

B. Identify referents for angles.

C. Sketch a given angle.

D. Estimate the measure of a given angle, using 22.5^0, 30^0, 45^0, 60^0, 90^0 and 180^0 as referent angles.

E. Measure, using a protractor, angles in various orientations.

F. Explain and illustrate how angles can be replicated in a variety of ways; *e.g.,* Mira, protractor, compass and straightedge, carpenter's square, dynamic geometry software.

G. Replicate angles in a variety of ways, with and without technology.

H. Bisect an angle, using a variety of methods.

I. Solve a contextual problem that involves angles.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.1 (A D E I)

5.2 (B C F G H I)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: G6 – Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by:

- **drawing;**
- **replicating and constructing;**
- **bisecting;**
- **solving problems.**

[C, ME, PS, T, V]

Elaboration

An angle is the figure formed by two rays sharing a common endpoint, called the vertex of the angle. Angles are classified according to their measure, as follows:

- **acute angle –** an angle measuring between 0^0 and 90^0
- **right angle –** an angle measuring 90^0
- **obtuse angle –** an angle measuring between 90^0 and 180^0
- **straight angle –** an angle measuring 180^0
- **reflex angle –** an angle measuring between 180^0 and 360^0

NUMBER

SPECIFIC CURRICULUM OUTCOMES

N1 – Solve problems that involve unit pricing and currency exchange, using proportional reasoning.

N2 – Demonstrate an understanding of income, including:
- **wages;**
- **salary;**
- **contracts;**
- **commission;**
- **piecework.**

to calculate gross and net pay.

MAT421K – Topic: Number (N)

GCO: Develop number sense and critical thinking skills.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
N1 Solve problems that involve unit pricing and currency exchange, using proportional reasoning.	**A3** Solve problems by applying proportional reasoning and unit analysis.

SCO: **N1 – Solve problems that involve unit pricing and currency exchange, using proportional reasoning.** [CN, ME, PS, R]

Students who have achieved this outcome should be able to:

A. Compare the unit price of two or more given items.

B. Solve problems that involve determining the best buy, and explain the choice in terms of the cost as well as other factors, such as quality and quantity.

C. Compare, using examples, different sales promotion techniques, *e.g.,* deli meat at $2 per 100 g seems less expensive than $20/kg.

D. Determine the percent increase or decrease for a given original and new price.

E. Solve, using proportional reasoning, a contextual problem that involves currency exchange.

F. Explain the difference between the selling rate and purchasing rate for currency exchange.

G. Explain how to estimate the cost of items in Canadian currency while in a foreign country, and explain why this may be important.

H. Convert between Canadian currency and foreign currencies, using formulas, charts or tables.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (A B C D)

1.2 (E F G H)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: N1 – Solve problems that involve unit pricing and currency exchange, using proportional reasoning.
[CN, ME, PS, R]

Elaboration

The essence of currency exchange is simple. When you travel, you have to exchange your money for the local currency. For example, in Europe you'll exchange Canadian dollars for Euros at the current exchange rate before you go shopping. This is the most familiar type of currency trade to most people.

Currencies are traded in pairs, each with its own exchange rate. An example would be exchanging Canadian dollars for U.S. dollars. The exchange rate might be reported as a number like 1.0522 – meaning it takes 1.0522 Canadian dollars to buy 1 U.S dollar. Conversely, it would take $\frac{1}{1.0522}$, or 0.9504 U.S. dollars to buy one Canadian dollar. The smallest increment by which a given pair can change by is called the pip. For the Canadian-U.S. dollar pair, that value is 0.0001, or $\frac{1}{100}$ of a cent.

In any transaction, the buyer will make an offer (bid) and the seller a counter offer (ask). For currency wholesalers, the bid/ask spread is 1 to 2 pips. When you trade a currency, your broker or bank marks the spread up to 3 to 20 pips (this is how the broker or bank makes its money). If you later sell the currency at an exchange rate that exceeds the spread, you make a profit.

For the current Canadian exchange rates, go to the dally currency converter at the Bank of Canada's website. It is found at http://www.bank-banque-canada.ca/en/rates/converter.html.

MAT421K – Topic: Number (N)

GCO: Develop number sense and critical thinking skills.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
N2 Demonstrate an understanding of income, including: • wages; • salary; • contracts; • commission; • piecework to calculate gross and net pay.	**N2** Solve problems that involve personal budgets.

SCO: **N2 – Demonstrate an understanding of income, including**

- **wages;**
- **salary;**
- **contracts;**
- **commission;**
- **piecework**

to calculate gross and net pay.
[C, CN, R, T]

Students who have achieved this outcome should be able to:

A. Describe, using examples, various methods of earning income.

B. Identify and list jobs that commonly use different methods of earning income; *e.g.,* hourly wage, wage and tips, salary, commission, contract, bonus, shift premiums.

C. Determine in decimal form, from a time schedule, the total time worked in hours and minutes, including time and a half and/or double time.

D. Determine gross pay from given or calculated hours worked when given:
- the base hourly wage, with and without tips;
- the base hourly wage, plus overtime (time and a half, double time).

E. Determine gross pay for earnings acquired by:
- base wage, plus commission;
- single commission rate.

F. Explain why gross pay and net pay are not the same.

G. Determine the Canadian Pension Plan (CPP), Employment Insurance (EI) and income tax deductions for a given gross pay.

H. Determine the net pay when given deductions, *e.g.,* health plans, uniforms, union dues, charitable donations, payroll tax.

I. Investigate, with technology, “what if …” questions related to changes in income, *e.g.,* “What if there is a change in the rate of pay?”

J. Identify and correct errors in a solution to a problem that involves gross or net pay.

K. Describe the advantages and disadvantages for a given method or earning income; *e.g.*, hourly wage, tips, piecework, salary, commission, contract work.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.1 (A B C D I J)
4.2 (F G H J)
4.3 (B E K)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: N2 – Demonstrate an understanding of income, including
- **wages;**
- **salary;**
- **contracts;**
- **commission;**
- **piecework**

to calculate gross and net pay.
[C, CN, R, T]

Elaboration

The focus of this outcome is to give students a clear understanding of the difference between gross and net pay. Focus should be placed on the various methods of earning a wage when determining a person's gross pay, such as earning an annual salary, earning an hourly wage, tips, straight commission, base salary plus commission, graduated commission, and piecework. The discussion should include how gross income is calculated in each case.

In determining a person's net pay, particular attention should be paid to the various deductions that an employee is subject to, such as income tax, Canada Pension Plan, employment insurance, and other deductions.

ALGEBRA

SPECIFIC CURRICULUM OUTCOMES

A1 – Solve problems that require the manipulation and application of formulas related to:

- **perimeter;**
- **area;**
- **the Pythagorean theorem;**
- **primary trigonometry ratios;**
- **income.**

MAT421K – Topic: Algebra (A)

GCO: Develop algebraic reasoning.

GRADE 10 – MAT421K	GRADE 11 – MAT521K
A1 Solve problems that require the manipulation and application of formulas related to: • perimeter; • area; • the Pythagorean theorem; • primary trigonometric ratios; • income.	**A1** Solve problems that require the manipulation and application of formulas related to: • volume and capacity; • surface area; • slope and rate of change; • simple interest; • finance charges.

SCO: **A1 – Solve problems that require the manipulation and application of formulas related to:**

- **perimeter;**
- **area;**
- **the Pythagorean theorem;**
- **primary trigonometry ratios;**
- **income.**

[C, CN, ME, PS, R]

Students who have achieved this outcome should be able to:

A. Solve a contextual problem that involves the application of a formula that does not require manipulation.

B. Solve a contextual problem that involves the application of a formula that requires manipulation.

C. Explain and verify why different forms of the same formula are equivalent.

D. Describe, using examples, how a given formula is used in a trade or an occupation.

E. Create and solve a contextual problem that involves a formula.

F. Identify and correct errors in a solution to a problem that involves a formula.

Note: It is intended that this outcome be integrated throughout the course.

Section(s) in Math at Work 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

Integrated throughout the text.

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: A1 – Solve problems that require the manipulation and application of formulas related to:
- **perimeter;**
- **area;**
- **the Pythagorean theorem;**
- **primary trigonometry ratios;**
- **income.**

[C, CN, ME, PS, R]

Elaboration

This particular outcome is integrated throughout the course. Specifically, students will be expected to manipulate formulas, such as:

Converting Fahrenheit Temperature to Celsius	$C = \frac{5}{9}(F - 32)$
Converting Celsius Temperature to Fahrenheit	$F = \frac{9}{5}C + 32$
Perimeter of a Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference of a Circle	$C = 2\pi r$ or $C = \pi d$
Area of a Rectangle	$A = lw$
Area of a Triangle	$A = \frac{bh}{2}$
Area of a Circle	$A = \pi r^2$
Surface Area of a Rectangular Prism	$SA = 2(lw + lh + wh)$
Surface Area of a Cylinder	$SA = 2\pi r^2 + 2\pi rh$
Pythagorean Theorem	$c^2 = a^2 + b^2$
Sine of Angle *A*	$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$
Cosine of Angle *A*	$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$
Tangent of Angle *A*	$\tan A = \frac{\text{opposite}}{\text{adjacent}}$
Gross Salary	$S = (\text{hourly rate}) \times (\text{hours worked})$

Curriculum Guide Supplement

This supplement to the *Prince Edward Island MAT421K Mathematics Curriculum Guide* is designed to parallel the primary resource, *Math at Work 10*.

For each of the chapters in the text, an approximate timeframe is suggested to aid teachers with planning. The timeframe is based on a total of 80 classes, each with an average length of 75 minutes:

CHAPTER	SUGGESTED TIME
Chapter 1 – Consumerism and Travel	11 classes
Chapter 2 – Measuring Length	13 classes
Chapter 3 – Measuring Area	12 classes
Chapter 4 – Getting Paid for Your Work	10 classes
Chapter 5 – All About Angles	13 classes
Chapter 6 – Pythagorean Relationship	8 classes
Chapter 7 – Trigonometry	13 classes

Each chapter of the text is divided into a number of sections. In this document, each section is supported by a one-page presentation, which includes the following information:

- the name and pages of the section in *Math at Work 10*;
- the specific curriculum outcome(s) and achievement indicator(s) addressed in the section (see the first half of the curriculum guide for an explanation of symbols);
- the student expectations for the section, which are associated with the SCO(s);
- the new concepts introduced in the section;
- other key ideas developed in the section;
- suggested problems in *Math at Work 10*;
- possible instructional and assessment strategies for the section.

CHAPTER 1
CONSUMERISM AND TRAVEL

SUGGESTED TIME

11 classes

Section 1.1 – Unit Pricing (pp. 6-17)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **N1 (A B C D)** **After this lesson, students will be expected to:** • calculate unit price • compare unit prices of two or more items • determine the best buy • analyse sales techniques • determine percent changes in prices **After this lesson, students should understand the following concept:** • **unit price –** the price for one unit of an item; examples include \$2.25/litre, \$5.90/metre, 50¢/apple **Suggested Problems in *Math at Work 10*:** • **pp. 10-11:** #1-8 • **pp. 14-15:** #1-9 • **pp. 16-17:** #1-7	**Possible Instructional Strategies:** • When calculating unit price, the money amount is always placed in the numerator of the rate. After dividing, the units will be written in the form dollars/unit. • When working with percents, it is often simplest to work with them in decimal form. To find the percent of a number, convert to a decimal and then multiply by the number. • An alternate way to calculate a discounted price is to subtract the percent discount from 100% and multiply the difference by the original price. • Remind students that the amount of discount and the discounted price must add up to the original price. **Possible Assessment Strategies:** • Calculate the unit price of each product. Round off each answer to the nearest cent. a. \$1.99 for 250 sheets of paper b. \$3.99 for 6 bottles of cola • A 20-ounce bag of popcorn costs \$2.80. If the unit price stays the same, how much does a 35-ounce bag cost? • Which is the better buy: 10 pencils for \$4.00 or 6 pencils for \$2.70? • Mr. Scrub offers three ways to pay for car washes: a book of six car wash coupons for \$33, a special offer of two washes for \$11.50, or one wash for \$5.95. Which option offers the least expensive unit price for one car wash? • The usual price for a movie ticket at Big Screen Cinemas is \$18. On Tuesdays, they offer a 15% discount. Calculate the cash value of the discount and the cost of a ticket on Tuesdays. • A quality pen that normally costs \$20 is being sold for only \$12. Calculate the discount in dollars, and also as a percentage of the marked price. • Ned got a 12% discount when he bought his new jacket. If the original price, before the discount, was \$50, how much did he pay for the jacket? • Sue is paid \$32 for 4 hours of babysitting. What rate does she get paid per hour?

Section 1.2 – Currency Exchange (pp. 18-28)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **N1 (E F G H)** **After this lesson, students will be expected to:** • convert between Canadian currency and foreign currencies • estimate the cost of items from another country in Canadian currency **After this lesson, students should understand the following concepts:** • **exchange rate –** a rate that specifies how much one currency is worth in terms of another; also known as the foreign-exchange rate • **proportion –** an equation that says two rates or ratios are equal; for example, $\frac{1}{4}=\frac{4}{16}$ **Suggested Problems in *Math at Work 10*:** • **pp. 22-23:** #1-10 • **pp. 26-27:** #1-8 • **pp. 27-28:** #1-6	**Possible Instructional Strategies:** • Remind students that currency exchange rates are constantly changing. The rates in the book are presented as examples that can be used for the purpose of this section. • Since an exchange rate tells you the price of one country's currency in terms of another, proportions can be used to determine the value of the amount exchanged. • For the current Canadian exchange rates, go to the daily currency converter at the Bank of Canada's website. It is found at http://www.bank-banque-canada.ca/en/rates/converter.html. **Possible Assessment Strategies:** • Kathy is planning a trip to Florida. She would like to exchange C$500.00 into American currency. How much American money will she get if the exchange rate is C$1 = US$1.0044? • Stephen would like to convert 350 British pounds into Canadian dollars. How much Canadian money will he get if the exchange rate is C$1 = £0.5010? Round off the answer to the nearest cent.

Section 1.3 – Measurement Comparisons (pp. 29-47)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **M1 (A B C D E F H)**
- **M2 (A B C D E F H)**

After this lesson, students will be expected to:

- convert between imperial units and SI units of mass, capacity, and temperature
- convert between imperial units of mass and capacity

After this lesson, students should understand the following concepts:

- **SI (Système international d'unités) –** a system of measurement in which units are based on powers of 10; also called the metric system of measurement
- **imperial system –** the system of measurement based on British units
- **ounce –** imperial unit of measure for weight; abbreviation is oz; $16 \text{ oz} = 1 \text{ lb}$
- **pound –** imperial unit of measure for weight; abbreviation is lb; $1 \text{ lb} = 16 \text{ oz}$
- **fluid ounce –** imperial unit of measure for capacity; abbreviation is fl oz; $128 \text{ fl oz} = 1 \text{ gal}$
- **gallon –** imperial unit of measure for capacity; abbreviation is gal; $1 \text{ gal} = 128 \text{ fl oz}$
- **Celsius –** a scale for measuring temperature in which the freezing point of water is 0^0C and the boiling point is 100^0C; abbreviation is ^{0}C
- **Fahrenheit –** a scale for measuring temperature in which the freezing point of water is 32^0F and the boiling point is 212^0F; abbreviation is ^{0}F
- **cup –** imperial unit of measure for capacity; abbreviation sometimes used is c; $1 \text{ cup} = 8 \text{ oz}$
- **quart –** imperial unit of measure for capacity; abbreviation is qt; $1 \text{ qt} = 4 \text{ cups}$

CONVERSION FACTORS			
SI UNIT	**IMPERIAL UNIT**	**SI UNIT**	**IMPERIAL UNIT**
1 kg	2.205 lb	1 lb	0.4536 kg
1 L	0.2642 gal	1 gal	3.785 L
1 mL	0.0338 fl oz	1 fl oz	29.574 mL

Suggested Problems in *Math at Work 10*:

- **pp. 34-35:** #1-8
- **p. 38:** #1-8
- **p. 41:** #1-7
- **pp. 44-45:** #1-8
- **pp. 46-47:** #1-8

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Have students establish referents for weights such as one ounce, one pound and one ton.
- Have students establish referents for masses such as one gram, one kilogram and one tonne.
- Ensure students know that 1 kilogram is approximately equivalent to 2.2 pounds.
- Ensure that students know how to work with both temperature conversion formulas:

$$C = \frac{5}{9}(F - 32)$$

$$F = \frac{9}{5}C + 32$$

- Students should memorize the freezing and boiling points of water in both the Celsius and Fahrenheit scales. These values can be used as benchmarks to help them determine if their answers are reasonable.

Possible Assessment Strategies:

- Convert 15^0C to degrees Fahrenheit.
- Convert 100^0F to degrees Celsius. Round off the answer to one decimal place.
- Determine the temperature at which degrees Celsius equals degrees Fahrenheit.
- Convert 5 lb to kilograms. Round off the answer to one decimal place.
- Convert 12 kg to pounds. Round off the answer to one decimal place.
- Convert 6 oz to grams. Round off the answer to the nearest gram.
- Convert 120 g to ounces. Round off the answer to one decimal place.
- Convert 25 L to gallons. Round off the answer to one decimal place.
- Convert 36 gal to litres. Round off the answer to one decimal place.
- Convert 56 cups to quarts.
- If one litre of gasoline costs 99.7 cents, how much will one gallon cost? Round off the answer to the nearest cent.

CHAPTER 2
MEASURING LENGTH

SUGGESTED TIME
13 classes

Section 2.1 – Imperial Length Measurements (pp. 58-69)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

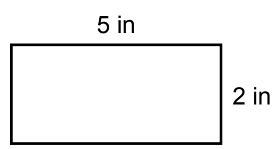
- **M2 (B C G)**
- **M3 (A B C D E)**

After this lesson, students will be expected to:

- describe the relationships among imperial units of length
- use referents to estimate length in imperial units
- calculate length in imperial units
- convert between imperial units for length

After this lesson, students should understand the following concepts:

- **inch –** a unit of length in the imperial system; $12 \text{ in} = 1 \text{ ft}$; the abbreviation is in or ″
- **foot –** the basic unit of length in the imperial system; the abbreviation is ft or ′
- **yard –** a unit of length in the imperial system; $1 \text{ yd} = 3 \text{ ft}$; the abbreviation is yd
- **perimeter –** the distance around the outside of an object; the symbol for perimeter is P



$$P = 2 + 5 + 2 + 5 = 14 \text{ in}$$

- **dimensions –** measurements such as the length, width, or height of an object

Suggested Problems in *Math at Work 10*:

- **pp. 62-63:** #1-10
- **pp. 66-67:** #1-11
- **pp. 68-69:** #1-9

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students remember the formula for the perimeter of a rectangle:

$$P = 2l + 2w \text{ or } P = 2(l + w)$$

Possible Assessment Strategies:

- Express each answer in feet and inches.
  - a. 38 in
  - b. $2 \text{ ft } 1 \text{ in} + 14 \text{ in}$
- Convert each of the following measurements:
  - a. 1242 feet to yards.
  - b. $3\frac{1}{4}$ feet to inches.
  - c. 21 inches to feet.
- Henry requires 15 feet of cable in order to install digital television in a house. If he has a spool with 300 yards of cable in his van, how many installations can he do with this spool of cable?
- Air Canada will accept carry-on luggage if its length, width and height add up to no more than 46 inches. Sean's suitcase has a length of 2 ft 3 in, width 1 ft 3 in, and height 10 inches. Will Air Canada allow him to carry his suitcase on board?

Section 2.2 – SI Length Measurements (pp. 70-81)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

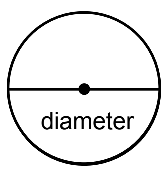
- **M1 (B C G)**
- **M3 (A B C D E)**

After this lesson, students will be expected to:

- describe the relationships among SI units of length
- use referents to estimate lengths in SI units
- calculate lengths in SI units
- convert between SI units for length

After this lesson, students should understand the following concept:

- **diameter –** the distance across the centre of a circle; the abbreviation is *d*



Suggested Problems in *Math at Work 10*:

- **pp. 74-75:** #1-9
- **pp. 78-79:** #1-11
- **pp. 80-81:** #1-8

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

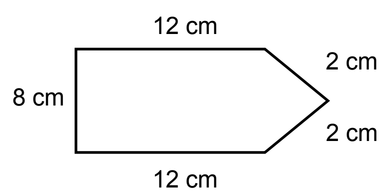
Possible Instructional Strategies:

- Remind students of the meaning of the SI prefixes:

PREFIX	ABBREVIATION	MULTIPLYING FACTOR
kilo-	k	1000
hecto-	h	100
deca-	da	10
deci-	d	0.1
centi-	c	0.01
milli-	m	0.001

Possible Assessment Strategies:

- Complete each of the following conversions.
  - a. 6 cm = ☐ mm
  - b. 12 m = ☐ cm
  - c. 3.7 dm = ☐ m
  - d. 1.34 m = ☐ mm
  - e. 7000 m = ☐ km
- What is the perimeter of the following shape?



Section 2.3 – Length Conversions (pp. 82-93)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **M1 (F H)**
- **M2 (F H)**

After this lesson, students will be expected to:

- approximate measurements between SI and imperial units for length
- convert between SI and imperial units for length
- solve problems that involve conversions between SI and imperial units for length

CONVERSIONS BETWEEN SI AND IMPERIAL UNITS	
SI UNITS TO IMPERIAL UNITS	**IMPERIAL UNITS TO SI UNITS**
1 mm ≐ 0.03937 in	1 in = 2.54 cm
1 cm ≐ 0.3937 in	1 ft = 30.48 cm
1 m ≐ 39.37 in 1 m ≐ 3.281 ft	1 yd = 91.44 cm 1 yd = 0.9144 m
1 km ≐ 0.6214 mi	1 mi ≐ 1.609 km

Suggested Problems in *Math at Work 10*:

- **pp. 86-87:** #1-11
- **pp. 90-91:** #1-8
- **pp. 92-93:** #1-8

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Remind students that when setting up a conversion factor, the numerator will include the unit to which he or she wants to convert. The denominator will be stated in the original units in which the measurement was taken.
- Students are not expected to memorize the conversion factors between imperial and SI units. They should be given a conversion table when solving conversion problems.

Possible Assessment Strategies:

- Convert 5 in to centimetres.
- Convert 15 cm to inches. Round off the answer to one decimal place.
- Convert 5 miles to kilometres. Round off the answer to one decimal place.
- Concert 32 km to miles. Round off the answer to one decimal place.
- A low bridge has a posted maximum vehicle height of 8 ft 6 in. Your truck is 2.5 m high. Will it fit under the bridge?
- Chris is 162 cm tall. Convert her height to feet and inches. Round off the answer to the nearest inch.
- It is 71 km from Summerside to Charlottetown. Convert this distance to miles. Round off the answer to the nearest mile.

Section 2.4 – Working with Length (pp. 94-105)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

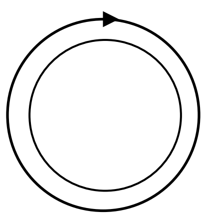
- **M3 (C D E F G H I)**

After this lesson, students will be expected to:

- calculate the circumference of a circle
- work with formulas using SI and imperial units of length
- solve and verify problems that involve formulas related to perimeter

After this lesson, students should understand the following concept:

- **circumference –** the distance around a circle; the perimeter of a circle; the abbreviation is *C*



Suggested Problems in *Math at Work 10*:

- **pp. 98-99:** #1-10
- **pp. 102-103:** #1-9
- **pp. 104-105:** #1-9

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

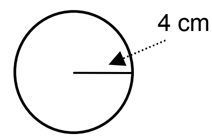
- Ensure that students remember the formulas for the circumference of a circle:

$$C = 2\pi r \text{ or } C = \pi d$$

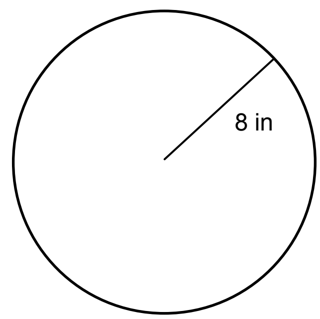
Possible Assessment Strategies:

- Determine the circumference of each circle. Round off the answers to one decimal place.

  a.



  b.



- Rob is putting a fence around a rectangular field that measures 25 m by 40 m. How much fencing will he need to enclose the field?
- Determine the midpoint of each length measurement.

  a. 7 in

  b. 82 cm

  c. $8\frac{1}{4}$ m

  d. $1\frac{3}{4}$ ft

CHAPTER 3
MEASURING AREA

SUGGESTED TIME
12 classes

Section 3.1 – Imperial Area Measurements (pp. 116-127)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **M4 (A B C D E G H K)**

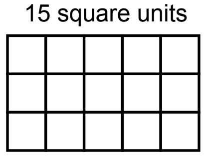
After this lesson, students will be expected to:

- estimate and calculate area in imperial units
- identify when to use an imperial unit of area
- solve problems that involve applying formulas
- determine whether a solution to a problem is reasonable

After this lesson, students should understand the following concept:

- **area –** the number of square units needed to cover the surface of a shape; abbreviation is *A*

15 square units



Suggested Problems in *Math at Work 10*:

- **pp. 120-121:** #1-8
- **pp. 124-125:** #1-9
- **pp. 126-127:** #1-9

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Remind students that when converting from one unit squared to another unit squared, they have to apply the conversion factor twice.
- Ensure that students remember the formula for the area of a rectangle:

$$A = lw$$

- Ensure that students remember the formula for the area of a triangle:

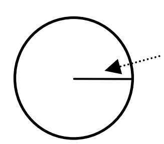
$$A = \frac{bh}{2}$$

- Ensure that students remember the formula for the area of a circle:

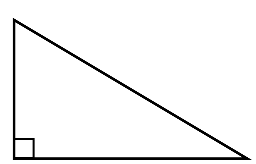
$$A = \pi r^2$$

Possible Assessment Strategies:

- Complete each of the following conversions.
  a. $3\ \text{ft}^2 = \square\ \text{in}^2$
  b. $\frac{1}{4}\ \text{ft}^2 = \square\ \text{in}^2$
  c. $576\ \text{in}^2 = \square\ \text{ft}^2$
  d. $18\ \text{ft}^2 = \square\ \text{yd}^2$
- John wants to paint a wall that measures 12 ft by 20 ft. One small can of paint will cover 100 ft^2 of area. At least how many cans of paint will he need to buy to paint the wall?
- Determine the area of the following circle. Round off the answer to one decimal place



- Determine the area of the following triangle.



Section 3.2 – SI Area Measurements (pp. 128-139)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **M4 (A B C D E F H K)**

After this lesson, students will be expected to:

- estimate and calculate area in SI units
- identify when to use an SI unit of area
- solve problems that involve applying formulas
- determine whether the solution to a problem is reasonable

FORMULAS FOR AREA	
Rectangle	$A = lw$
Triangle	$A = \frac{bh}{2}$
Circle	$A = \pi r^2$

Suggested Problems in *Math at Work 10*:

- **pp. 132-133:** #1-10
- **pp. 136-137:** #1-8
- **pp. 138-139:** #1-8

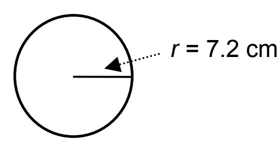
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

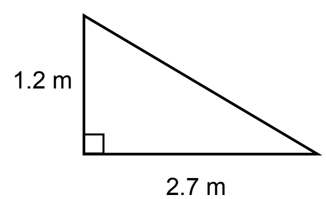
- Remind students that when converting from one unit squared to another unit squared, they have to apply the conversion factor twice.

Possible Assessment Strategies:

- Complete each of the following conversions.
  a. $2\ m^2 = \square\ cm^2$
  b. $0.4\ m^2 = \square\ cm^2$
  c. $800\ mm^2 = \square\ cm^2$
  d. $5{,}000{,}000\ m^2 = \square\ km^2$
- Determine the area of the following circle. Round off the answer to one decimal place.



- Determine the area of the following triangle.



- A pathway is 24 m long and 1.2 m wide. It is to be covered with bricks that are 12 cm long and 6 cm wide. How many bricks will be needed to cover the pathway?

Section 3.3 – Working with Area (pp. 140-151)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

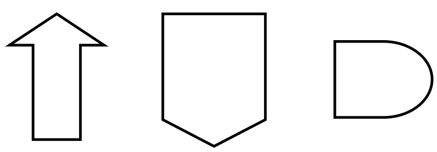
- **M4 (D E J)**

After this lesson, students will be expected to:

- estimate the area of a composite 2-D shape using a grid
- solve problems that involve the area of composite 2-D shapes
- explain how changing one or more dimensions affects the area of a rectangle

After this lesson, students should understand the following concept:

- **composite shape –** a shape made of two or more shapes



Suggested Problems in *Math at Work 10*:

- **pp. 144-145:** #1-6
- **pp. 148-149:** #1-5
- **pp. 150-151:** #1-7

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

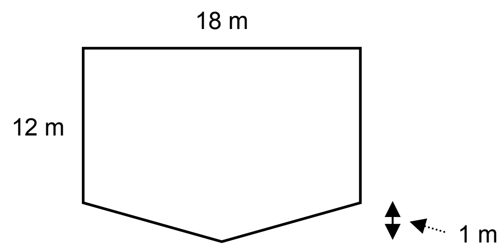
Possible Instructional Strategies:

- Remind students that when trying to find the area of a composite shape, that it should be split up into its different parts.

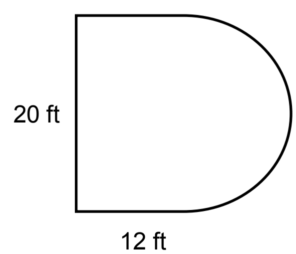
Possible Assessment Strategies:

- Determine the area of each composite shape. Round off the answers to one decimal place, where necessary.

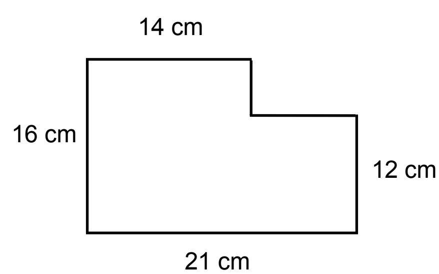
a.



b.



c.



- The length and the width of a rectangle are doubled. How does the area of the larger rectangle compare to the area of the smaller rectangle?

Section 3.4 – Surface Area of Three-Dimensional Objects (pp. 152-163)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

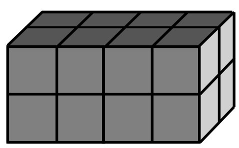
- **M4 (I)**

After this lesson, students will be expected to:

- display the surface area of a 3-D object as a combination of 2-D shapes
- calculate the surface area of 3-D objects

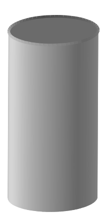
After this lesson, students should understand the following concepts:

- **surface area –** the number of square units needed to cover an object



- **rectangular prism –** a 3-D object in which the six sides are made up of three pairs of rectangles

- **right cylinder –** a 3-D object with two parallel circular bases



FORMULAS FOR SURFACE AREA	
Rectangular Prism	$SA = 2(lw + lh + wh)$
Cylinder	$SA = 2\pi r^2 + 2\pi rh$

Suggested Problems in *Math at Work 10*:

- **pp. 156-157:** #1-5
- **pp. 160-161:** #1-7
- **pp. 162-163:** #1-6

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Encourage students to draw the net of a three-dimensional object when determining its surface area.

Possible Assessment Strategies:

- What is the surface area of a wood box with length 5 ft, width 3 ft, and height 2 ft?
- The surface area of a cube is 216 cm^2. Find the length of one side of the cube.
- A can of pop is 9 cm high and its top has a radius of 4 cm. What is its total surface area? Round off the answer to one decimal place.
- A company manufactures aluminum beverage cans in two sizes: Can A has a diameter of 2.5 in and a height of 4.5 in. Can B has a diameter of 3 in and a height of 3 in. Which can requires more aluminum to manufacture?

CHAPTER 4
GETTING PAID FOR YOUR WORK

SUGGESTED TIME
10 classes

Section 4.1 – Wages and Salary (pp. 174-185)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **N2 (A B C D I J)** **After this lesson, students will be expected to:** • identify jobs that can earn an hourly wage, wage and tips, or a salary • determine the total time worked • determine gross pay, including time-and-a-half and double time • investigate questions related to changes in pay • identify and correct errors in gross pay **After this lesson, students should understand the following concepts:** • **salary –** a fixed amount of money paid to a person on a monthly or annual basis; does not depend on the number of hours worked • **hourly wage –** the amount of money paid to a worker per hour worked • **gross pay –** total earnings, from a salary, hourly wage, or other payment method, such as tips • **overtime –** payment for work done in addition to regular hours; overtime is usually equal to 1.5 times regular pay, but it can be more • **shift premium –** an additional payment made for working undesirable shifts, such as overnight shifts **Suggested Problems in *Math at Work 10*:** • **pp. 178-179:** #1-8 • **pp. 182-183:** #1-7 • **pp. 184-185:** #1-7	**Possible Instructional Strategies:** • Ask students to research the minimum wage of each Canadian province. Which province has the highest and lowest minimum wage? • Remind students that being paid bi-weekly (26 times/year) and semi-monthly (24 times/year) are not the same. **Possible Assessment Strategies:** • Paul's regular rate of pay is $9.40 per hour and he works 25 hours per week. What is his gross pay per week? • Jennie worked for 38 hours and earned $446.50. What is her hourly wage? • Jim earns $39,000 per year. If he is paid bi-weekly, what is his gross pay each pay week? • Jeanne works full time in a clothing store. Her regular work week is 38 hours and she earns $8.50 per hour. If she works more than 38 hours a week, she earns time and a half for the extra hours. One week, she works the hours listed below. What will Jeanne's gross pay for that week be? Day / Hours: Monday 7; Tuesday 7; Wednesday 8; Thursday 8; Friday 8; Saturday 4 • Calculate the time-and-a-half rate for each of the following hourly wages. a. $12.50 b. $18.90 c. $10.00 • Mary earns $8.70 per hour. She is paid time and a half for overtime hours over 40 hours. How much will she earn for the week if she works 44 hours? • Glen works at a car manufacturing plant. His regular work hours are 9:00 A.M. to 5:00 P.M from Monday to Friday. At this plant, a shift premium of $5.00 per hour is paid on any hours worked between 3:00 P.M. and 5:00 P.M. If his regular hourly rate of pay is $25.50 per hour, how much is his gross pay for a week?

Day	Hours
Monday	7
Tuesday	7
Wednesday	8
Thursday	8
Friday	8
Saturday	4

Section 4.2 – Net Pay (pp. 186-197)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **N2 (F G H J)** **After this lesson, students will be expected to:** • identify the difference between gross pay and net pay • determine deductions and net pay • determine CPP, EI, and income tax deductions for a given gross pay • identify and correct errors involving gross pay **After this lesson, students should understand the following concepts:** • **deduction –** an amount of money subtracted from gross pay; includes income tax, Canada Pension Plan (CPP), and Employment Insurance (EI); can also include union dues, and health or dental insurance • **net pay –** the amount you receive on a paycheque after deductions have been taken off; also called take-home pay **Suggested Problems in *Math at Work 10*:** • **p. 190:** #1-5 • **pp. 194-195:** #1-9 • **pp. 196-197:** #1-8	**Possible Instructional Strategies:** • Remind students that an employee's CPP, EI and income tax deduction depends on his or her salary. • Actual CPP and EI rates vary from year to year. Use the values given in the textbook for the problems in this section. • Income tax tables for each province can be found at http://www.cra-arc.gc.ca/. **Possible Assessment Strategies:** • Bob Sloan's weekly salary is $570.00. Use 52 pay periods in a year to answer these questions. a. Calculate his weekly CPP deduction if the contribution rate is 4.95% of any annual gross earnings above $3500.00. Round off the answer to the nearest cent. b. Calculate his weekly EI deduction if the EI premium rate is 1.73% of gross earnings. Round off the answer to the nearest cent. • Paula DuMont works a 40-hour work week and earns $11.25 per hour with time and a half for overtime. Over the last two weeks, she has worked 45 hours each week. Calculate her net pay for the two weeks if all of her deductions add up to 35% of her gross pay. Round off the answer to the nearest cent. • Stuart works as a waiter and earns $800.00 bi-weekly. Each pay period, his employer deducts $59.50 for federal tax, $16.91 for provincial tax, and 1.73% for EI premiums. The CPP contribution rate is 4.95% of any annual gross earnings above $3500.00. What is Stuart's bi-weekly net pay? Use 26 pay periods for the year. Round off the answer to the nearest cent.

Section 4.3 – Other Forms of Income (pp. 198-209)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **N2 (B E K)** **After this lesson, students will be expected to:** • identify types of employment where commission is paid • determine gross pay for commission earnings • determine earnings for piecework and contract work • describe the advantages and disadvantages of different methods of earning income **After this lesson, students should understand the following concepts:** • **commission –** income based on amount of sales; often a percent of an item or service • **bonus –** an amount of money paid to an employee as a reward for higher sales or a job well done • **piecework –** an amount of money earned per unit of work; for example, a unit of work may be a planted tree, a completed piece of clothing, or a typed article • **contract –** an agreement between an employer and an employee; it states that a certain amount of work is to be completed within a set period of time for a set amount of pay **Suggested Problems in *Math at Work 10*:** • **pp. 202-203:** #1-8 • **pp. 206-207:** #1-7 • **pp. 208-209:** #1-8	**Possible Instructional Strategies:** • When calculating total earnings, ensure that students solve the problems in steps, and then find the total of all of the steps. • Remind students that when solving commission problems, the commission earned is added to the base salary. **Possible Assessment Strategies:** • Elena works part-time at an electronics store. Her weekly salary is $350 plus 5% commission on her total sales. Find her total sales when her total income for the week is $450. • Jason is paid $1200 per month plus a commission of 4% of the first $2,500 of his sales, and 6% of his sales over $2,500. Last month his sales totaled $22,100. Find Jason's wages for the month. • Candace gets an annual bonus of 18% of her base salary if she exceeds her sales quota by $25,000. If her base salary is $56,000 and she exceeds her quota by the required amount, what will be her salary for that year? • John works as a bus boy in a restaurant and earns 2% of all tips earned during his shift. If $1567.00 were collected in tips, how much will John's portion be? • Jake works at a call centre promoting low-rate credit cards. He earns $10.50 per hour plus $10 for every sale he makes. This past week, he worked 37.5 hours and was able to sign up 36 customers. What was his gross pay for the week?

CHAPTER 5
ALL ABOUT ANGLES

SUGGESTED TIME

13 classes

Section 5.1 – Estimating and Measuring Angles (pp. 222-233)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

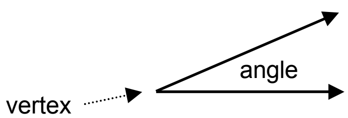
- **G6 (A D E I)**

After this lesson, students will be expected to:

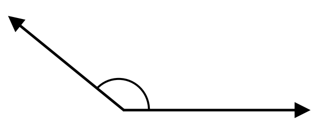
- estimate the measure of angles
- measure angles using a protractor
- classify types of angles

After this lesson, students should understand the following concepts:

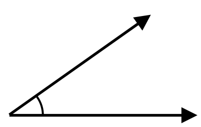
- **angle –** formed by two line segments that start from the same point, which is called a vertex; measured in degrees; the symbol for degrees is ^{0}



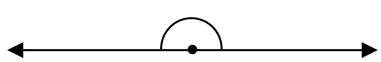
- **obtuse angle –** measures between 90^0 and 180^0



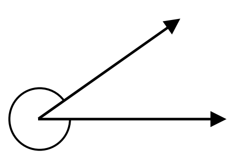
- **acute angle –** measures less than 90^0



- **straight angle –** measures exactly 180^0



- **reflex angle –** measures more than 180^0 but less than 360^0



Suggested Problems in *Math at Work 10*:

- **pp. 226-227:** #1-11
- **pp. 230-231:** #1-8
- **pp. 232-233:** #1-10

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Remind students that degrees can both represent a scale that can be used to measure angles or temperatures.
- A clock face is an excellent tool to use for angle referents.

Possible Assessment Strategies:

- Name the fraction of a turn that is represented by each angle.
  a. 45^0
  b. 120^0
  c. 270^0
- Name the type of angle that has each of the following measures.
  a. 100^0
  b. 45^0
  c. 182^0
  d. 90^0
  e. 180^0

Section 5.2 – Angle Constructions (pp. 234-245)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **G6 (B C F G H I)** **After this lesson, students will be expected to:** • sketch angles using a reference • construct angles • bisect angles **After this lesson, students should understand the following concepts:** • **bisect –** cut in half • **angle bisector –** line that cuts an angle into two equal pieces **Suggested Problems in *Math at Work 10*:** • **pp. 238-239:** #1-8 • **pp. 242-243:** #1-7 • **pp. 244-245:** #1-9	**Possible Instructional Strategies:** • Explain to students the purpose of each of the instruments in a geometry set when creating geometric constructions. **Possible Assessment Strategies:** • Determine the measure of the angle that bisects each of these angles. a. 116^0 b. 78^0 c. 35^0

Section 5.3 – Lines and Angles (pp. 246-261)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

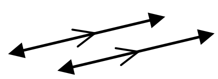
- **G5 (A B C D E F G)**

After this lesson, students will be expected to:

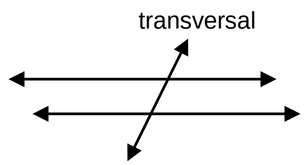
- identify perpendicular, parallel, and transversal lines
- identify patterns of angles formed by parallel lines
- identify patterns of angles when two lines cross

After this lesson, students should understand the following concepts:

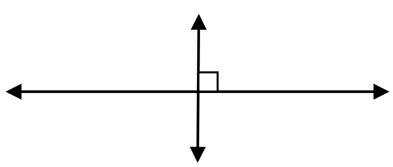
- **parallel lines –** lines that do not cross each other; are marked by matching arrowheads; two parallel lines are always the same distance apart



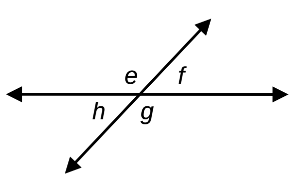
- **transversal –** a line that crosses two or more parallel lines



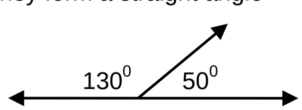
- **perpendicular lines –** lines that cross each other at right angles; often marked with one right angle symbol



- **opposite angles –** a pair of equal angles formed by two lines that cross; they form an X pattern; in the diagram below, $e = g$ and $f = h$



- **supplementary angles –** two angles that add up to 180^0; they form a straight angle



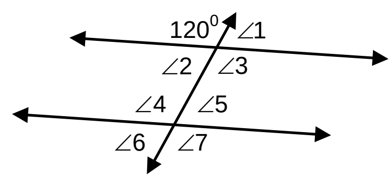
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Review with students the various pairs of angles such as vertically opposite, corresponding, alternate interior and alternate exterior angles.

Possible Assessment Strategies:

- Find the measure of the complementary angle to each of the following angles.
  a. 56^0
  b. 32^0
  c. 82^0
- Find the measure of the supplementary angle to each of the following angles.
  a. 126^0
  b. 38^0
  c. 90^0
- Find the measure of each of the indicated angles.



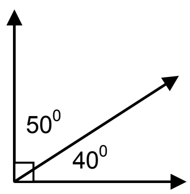
  a. $\angle 1$
  b. $\angle 2$
  c. $\angle 3$
  d. $\angle 4$
  e. $\angle 5$
  f. $\angle 6$
  g. $\angle 7$

Section 5.3 – Lines and Angles (Continued) (pp. 246-261)

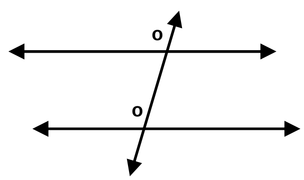
<table>
<tr><th>ELABORATIONS &
SUGGESTED PROBLEMS</th><th>POSSIBLE INSTRUCTIONAL &
ASSESSMENT STRATEGIES</th></tr>
<tr><td>

After this lesson, students should understand the following concepts:

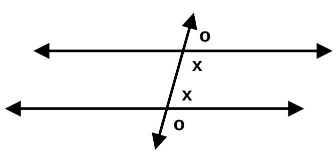
- **complementary angles –** two angles that add up to 90^0; they form a right angle; they form an L pattern



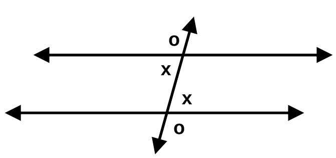
- **corresponding angles –** a pair of angles on the same side of the transversal crossing parallel lines; they are equal; they form an F pattern



- **same side exterior angles** – a pair of angles on the same side of a transversal and outside the parallel lines; they add up to 180^0; denoted by **o** in the diagram below



- **same side interior angles** – a pair of angles on the same side of a transversal and inside the parallel lines; they add up to 180^0; they form a C pattern; denoted by **x** in the diagram above
- **alternate exterior angles –** a pair of angles on opposite sides of a transversal and outside the parallel lines; they are equal; denoted by **o** in the diagram below



- **alternate interior angles –** a pair of angles on opposite sides of a transversal and inside the parallel lines; they are equal; they form a Z pattern; denoted by **x** in the diagram above

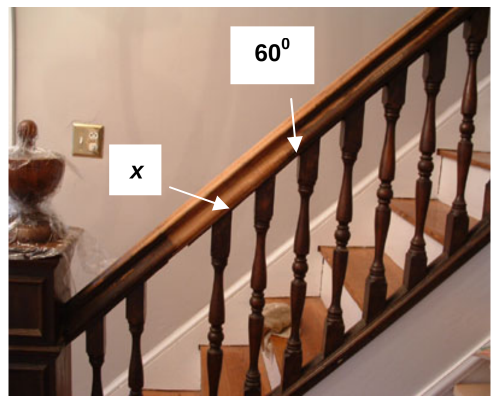
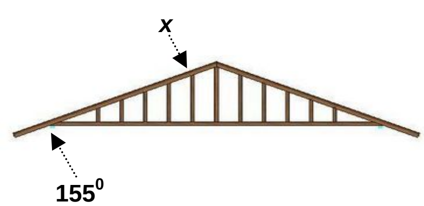
</td><td>

Suggested Problems in *Math at Work 10*:

- **pp. 250-251:** #1-7
- **pp. 254-255:** #1-8
- **pp. 258-259:** #1-7
- **pp. 260-261:** #1-8

</td></tr>
</table>

Section 5.4 – Angles in Our World (pp. 262-273)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **G5 (H I)** **After this lesson, students will be expected to:** • solve problems with angles formed by parallel lines and a transversal **Suggested Problems in *Math at Work 10*:** • **pp. 266-267:** #1-7 • **pp. 270-271:** #1-6 • **pp. 272-273:** #1-9	**Possible Instructional Strategies:** • Review with students the various pairs of angles such as vertically opposite, corresponding, alternate interior and alternate exterior angles. **Possible Assessment Strategies:** • To ensure that his grandmother can get upstairs safely, Robert is installing a new banister on her stairs. Vertical posts, called spindles, attached to the handrails of a banister must be parallel to each other. What is the measure of angle *x* in the picture?  • The overhang of this gable end truss has an angle of 155^0 with the horizontal. What is the measure of angle *x*? 

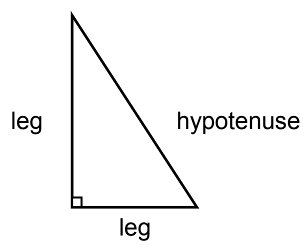
CHAPTER 6

PYTHAGOREAN RELATIONSHIP

SUGGESTED TIME

8 classes

Section 6.1 – Right Triangles (pp. 284-291)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **G2 (A)** **After this lesson, students will be expected to:** • explore right triangles • estimate the length of the hypotenuse of a right triangle **After this lesson, students should understand the following concepts:** • **right triangle –** a triangle with an angle of 90^0  • **hypotenuse –** the longest side of a right triangle; it is located opposite the right angle • **leg –** one of the two shorter sides that forms the right angle in a right triangle **Suggested Problems in *Math at Work 10*:** • **pp. 288-290:** #1-6 • **pp. 290-291:** #1-5	**Possible Instructional Strategies:** • Remind students that all right triangles have a 90^0 angle and that the hypotenuse is the side of the triangle opposite that angle. • Remind students that the longest side of a right triangle is the hypotenuse. **Possible Assessment Strategies:** • If a field of potatoes measures 30 m by 50 m, estimate how long the diagonal of that field would be.

Section 6.2 – The Pythagorean Relationship (pp. 292-302)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

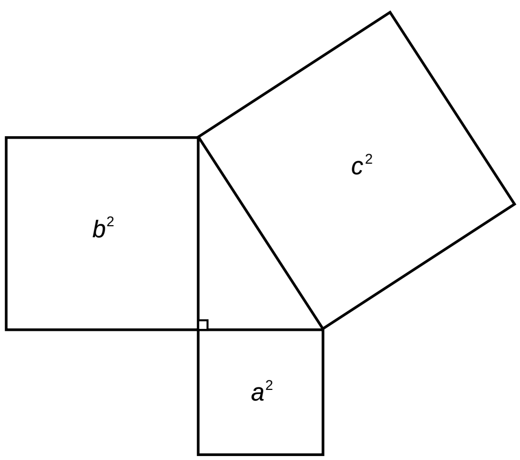
- **G2 (B C G)**

After this lesson, students will be expected to:

- verify the Pythagorean relationship
- determine the length of the hypotenuse of a right triangle
- determine the length of a leg of a right triangle

After this lesson, students should understand the following concept:

- **Pythagorean relationship –** the relationship among the lengths of the sides of a right triangle; the sum of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse



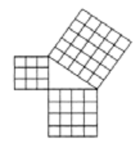
$a^2 + b^2 = c^2$

Suggested Problems in *Math at Work 10*:

- **pp. 296-297:** #1-7
- **pp. 299-300:** #1-6
- **pp. 301-302:** #1-10

Possible Instructional Strategies:

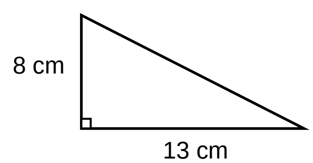
- Give or have students draw a variety of right triangles which have whole number sides, such as the 3-4-5, 6-8-10 or the 5-12-13 triangle. Have students cut out squares from centimetre grid paper so the sides of each square are the same as the side lengths for each triangle. Place the squares on the sides of each triangle as shown. Find the area of each square. Ask students what they notice.



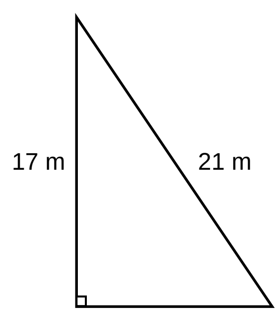
Possible Assessment Strategies:

- A right triangle has legs of length 5 cm and 7 cm. Between what two values must the length of the hypotenuse lie between?
- Determine the missing side of each right triangle. Round off the answers to one decimal place.

a.



b.



- Explain how you can determine whether or not a triangle is a right triangle if you know that it has side lengths of 7 cm, 11 cm and 15 cm.
- A 40-inch TV measures 34 inches along the bottom of the screen. What is the height of the screen. Round off the answer to the nearest inch.

Section 6.3 – Using the Pythagorean Relationship (pp. 303-313)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **G2 (D E F G)** **After this lesson, students will be expected to:** • show that a 3-4-5 triangle is a right triangle • determine if a triangle is a right triangle • determine if a corner is square using the Pythagorean relationship • solve problems involving right triangles **After this lesson, students should understand the following concept:** • **square –** a corner that is exactly 90^0 **Suggested Problems in *Math at Work 10*:** • **pp. 306-307:** #1-8 • **pp. 310-311:** #1-6 • **pp. 312-313:** #1-7	**Possible Instructional Strategies:** • Ensure that students make a clear distinction between the hypotenuse and the legs of a right triangle in any orientation. **Possible Assessment Strategies:** • Determine if there exists a right triangle with the following side lengths. a. 5, 12, 13 b. 7, 9, 11 c. 6, 8, 10 • John wonders if a picture frame is square. The horizontal pieces of the frame are 24 inches long. the vertical pieces are 18 inches long. He measures the diagonal and finds it to be 30 inches. Is his picture frame square? Explain.

CHAPTER 7

TRIGONOMETRY

SUGGESTED TIME

13 classes

Section 7.1 – Similarity and Scale (pp. 324-335)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

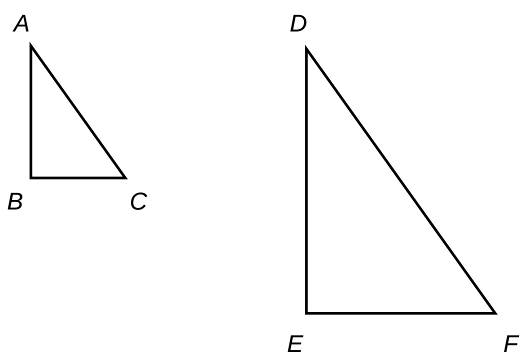
- **G3 (A B C D E F G)**

After this lesson, students will be expected to:

- determine if two triangles or two rectangles are similar
- determine if two irregular polygons are similar
- draw a polygon that is similar to a given polygon
- explain why two right triangles are similar
- solve problems involving similarity

After this lesson, students should understand the following concepts:

- **polygon –** a two-dimensional closed figure with edges that are line segments
- **similar figures –** have the same shape but different size; have equal corresponding angles; have proportional corresponding sides
- **corresponding sides –** sides that have the same relative position



For these triangles, the corresponding sides are:

- *AB* and *DE*
- *AC* and *DF*
- *BC* and *EF*

Suggested Problems in *Math at Work 10*:

- **pp. 328-329:** #1-9
- **pp. 332-333:** #1-6
- **pp. 334-335:** #1-9

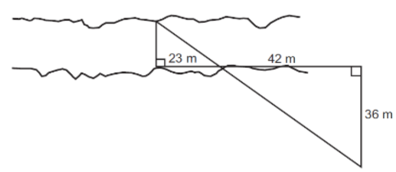
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

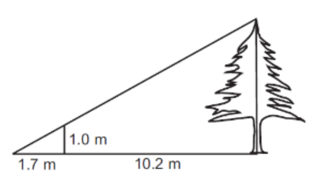
- Some students may benefit from tracing two similar triangles to compare angles and sides or to use tick marks or dashes to identify the corresponding angles and sides. This may help in setting up the ratios.
- Some students may find it helpful to redraw a diagram containing two triangles as two separate triangles. Students may also find it helpful to redraw the triangles so that the orientations are the same.

Possible Assessment Strategies:

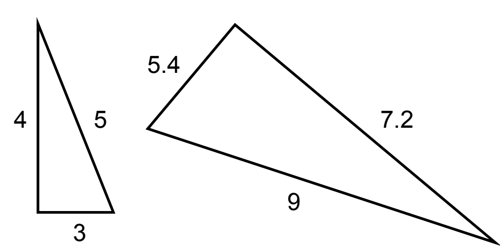
- A photograph measuring 12.5 cm by 17.5 cm needs to be enlarged by a ratio of 1.8. What will be the dimensions of the new photograph?
- A baseball coach wants to have the diagram of a baseball diamond that is similar to a real baseball diamond. A real baseball diamond is a square with side lengths of 90 ft. What will be the length of the side of the square in his diagram, if it is drawn to scale using a factor of 0.005? Give the answer in inches.
- The two triangles in the following diagram are similar. Determine the width of the river. Round off the answer to one decimal place.



- Use similar triangles to find the height of the tree.



- Determine if these two triangles are similar.



Section 7.2 – The Tangent Ratio (pp. 336-347)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

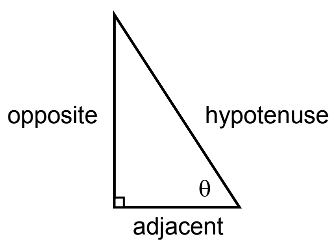
- **G4 (A D E F)**

After this lesson, students will be expected to:

- identify the hypotenuse, opposite side, and adjacent side in a right triangle
- show the relationship between the ratios of the side lengths in a set of similar right triangles
- define the tangent ratio in a right triangle
- use the tangent ratio to solve for an unknown side length in a right triangle

After this lesson, students should understand the following concepts:

- **opposite side –** the side across from the angle in a right triangle that you are working with



- **adjacent side –** the side beside the angle in a right triangle that you are working with which is not the hypotenuse
- **tangent ratio –** for an angle in a right triangle, the ratio of the length of the opposite side to the length of the adjacent side; the short form for the tangent ratio of $\angle A$ is tan *A*; $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

Suggested Problems in *Math at Work 10*:

- **pp. 340-341:** #1-9
- **pp. 344-345:** #1-7
- **pp. 346-347:** #1-10

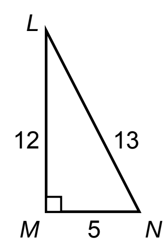
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

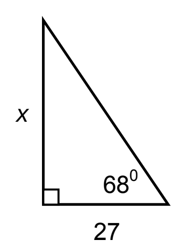
- Remind students that a diagram showing all the given information should be the first step in the solution of any trigonometric problem. Their sketches need not be accurate, but reasonable representations of the given situation can help them decide on a strategy.
- Some students have difficulty identifying the opposite and adjacent sides for the angle under consideration. Remind them that the hypotenuse is the longest side in a right triangle. Then, the adjacent side will be the leg of the right triangle that is next to the angle under consideration and the right angle, and the opposite side will be the leg opposite the angle under consideration.
- A mnemonic device can be used to help students such as ***T****own* ***O****f* ***A****lberton* for $\tan\,\theta = \frac{\text{opposite}}{\text{adjacent}}$.

Possible Assessment Strategies:

- For the given triangle, determine the value of each trigonometric ratio.
  - a. tan *L*
  - b. tan *N*

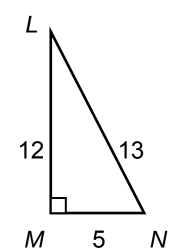
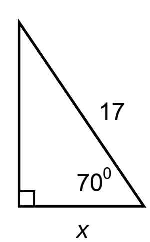


- Evaluate each trigonometric ratio, to four decimal places.
  - a. tan 63^0
  - b. tan 22^0
  - c. tan 49^0
- Find the length of side *x*. Round off the answer to one decimal place.

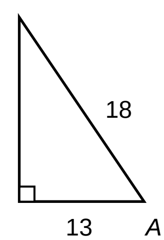


- A ladder leaning against a wall forms an angle of 63^0 with the ground. How far up the wall will the ladder reach if the foot of the ladder is 2 m from the wall? Round off the answer to one decimal place.

Section 7.3 – The Sine and Cosine Ratios (pp. 348-363)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **G4 (B C D E F)** **After this lesson, students will be expected to:** • show the relationships between the ratios of the side lengths in a set of similar triangles • define the sine ratio and the cosine ratio in a right triangle • use the sine ratio and the cosine ratio to solve for an unknown side length in a right triangle • determine which trigonometric ratio is most appropriate to solve a problem **After this lesson, students should understand the following concepts:** • **sine ratio –** for an angle in a right triangle, the ratio of the length of the opposite side to the length of the hypotenuse; the short form for the sine ratio of $\angle A$ is $\sin A$; $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ • **cosine ratio –** for an angle in a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse; the short form for the cosine ratio of $\angle A$ is $\cos A$; $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$ **Suggested Problems in *Math at Work 10*:** • **pp. 352-353:** #1-9 • **pp. 356-357:** #1-9 • **pp. 360-361:** #1-6 • **pp. 362-363:** #1-8	**Possible Instructional Strategies:** • Remind students that a diagram showing all the given information should be the first step in the solution of any trigonometric problem. Their sketches need not be accurate, but reasonable representations of the given situation can help them decide on a strategy. • Some students will get confused between the sine and cosine ratios. Mnemonic devices can be used to help students such as ***S**hips **O**f **H**alifax* for $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$. • Some students get confused between the sine and cosine ratios. Mnemonic devices can be used to help students such as ***C**anadian **A**mateur **H**ockey* for $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$. **Possible Assessment Strategies:** • For the given triangle, determine the value of each trigonometric ratio. a. sin *L* b. sin *N* c. cos *L* d. cos *N*  • Evaluate each trigonometric ratio, to four decimal places. a. $\sin 57^0$ b. $\cos 23^0$ • Find the length of side *x*. Round off the answer to one decimal place.  • A pilot starts his takeoff and climbs steadily at an angle of 12.2^0. Determine the distance, along the flight path, that the plane has travelled when it is at an altitude of 5.4 km. Express the answer to one decimal place.

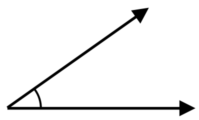
Section 7.4 – Determining Unknown Angles (pp. 364-375)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **G4 (E F)** **After this lesson, students will be expected to:** • solve problems that involve an unknown angle in a right triangle • use trigonometric ratios to solve problems **After this lesson, students should understand the following concept:** • **primary trigonometric ratios –** the three ratios sine, cosine, and tangent, defined in a right triangle **Suggested Problems in *Math at Work 10*:** • **pp. 368-369:** #1-8 • **pp. 372-373:** #1-11 • **pp. 374-375:** #1-7	**Possible Instructional Strategies:** • Ensure that students understand that a side and the angle opposite to that side in a right triangle have the same variable name, with the angle name written in uppercase and the side name written in lowercase. • Another mnemonic device that can be used to remember the three primary trigonometric ratios is **SOH – CAH – TOA.** **Possible Assessment Strategies:** • Determine the measure of each angle, to the nearest tenth of a degree. a. tan $\theta = 0.5123$ b. tan $\theta = 1.2079$ c. sin $\theta = 0.4384$ d. cos $\theta = 0.3178$ • Find the measure of angle *A*. Round off the answer to one decimal place.  • A water bomber is flying at an altitude of 5000 ft. The plane's radar shows that it is 8000 ft from the target, measured along the ground. What is the angle of elevation of the plane measured from the target site? Round off the answer to one decimal place. • A guy wire supporting a cell tower is 24 m long. If the wire is attached at a height of 17 m up the tower, determine the angle that the guy wire forms with the ground. Round off the answer to the nearest tenth of a degree. • A radio transmission tower is to be supported by a guy wire. The wire reaches 30 m up the tower and is attached to the ground a horizontal distance of 14 m from the base of the tower. What angle does the guy wire form with the ground, to the nearest tenth of a degree?

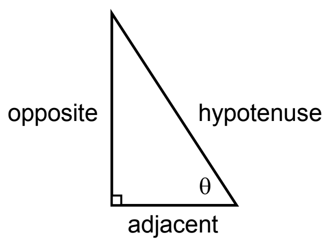
GLOSSARY OF MATHEMATICAL TERMS

A

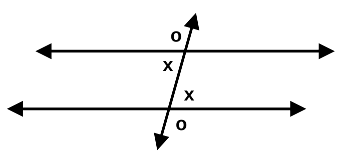
- **acute angle –** measures less than 90^0



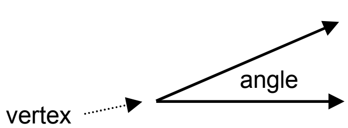
- **adjacent side –** the side beside the angle in a right triangle that you are working with which is not the hypotenuse



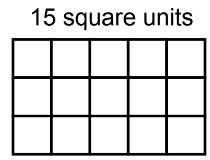
- **alternate exterior angles –** a pair of angles on opposite sides of a transversal and outside the parallel lines; they are equal; denoted by **o** in the diagram below



- **alternate interior angles –** a pair of angles on opposite sides of a transversal and inside the parallel lines; they are equal; they form a Z pattern; denoted by **x** in the diagram above
- **angle** – formed by two line segments that start from the same point, which is called a vertex; measured in degrees; the symbol for degrees is ^{0}



- **angle bisector –** line that cuts an angle into two equal pieces
- **area –** the number of square units needed to cover the surface of a shape; abbreviation is *A*

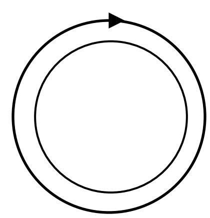


B

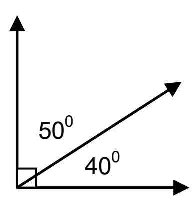
- **bisect –** cut in half
- **bonus –** an amount of money paid to an employee as a reward for higher sales or a job well done

C

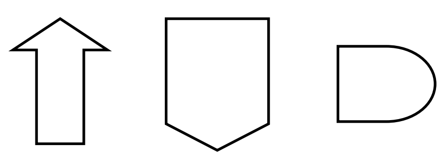
- **Celsius –** a scale for measuring temperature in which the freezing point of water is 0^0C and the boiling point is 100^0C; abbreviation is ^{0}C
- **circumference –** the distance around a circle; the perimeter of a circle; the abbreviation is *C*



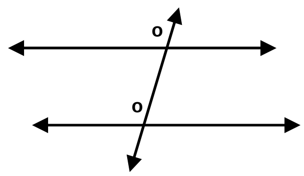
- **commission –** income based on amount of sales; often a percent of an item or service
- **complementary angles** – two angles that add up to 90^0; they form a right angle; they form an L pattern



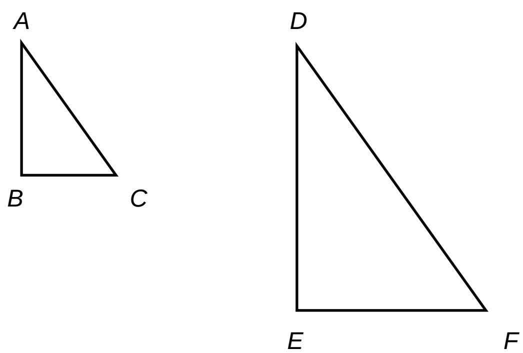
- **composite shape –** a shape made of two or more shapes



- **contract –** an agreement between an employer and an employee; it states that a certain amount of work is to be completed within a set period of time for a set amount of pay
- **corresponding angles –** a pair of angles on the same side of the transversal crossing parallel lines; they are equal; they form an F pattern



- **corresponding sides –** sides that have the same relative position



For these triangles, the corresponding sides are:
  - *AB* and *DE*
  - *AC* and *DF*
  - *BC* and *EF*

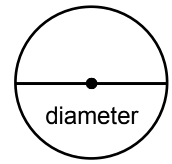
- **cosine ratio –** for an angle in a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse; the short form for the cosine ratio of $\angle A$ is $\cos A$;

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

- **cup –** imperial unit of measure for capacity; abbreviation sometimes used is c; $1 \text{ cup} = 8 \text{ oz}$

D

- **deduction –** an amount of money subtracted from gross pay; includes income tax, Canada Pension Plan (CPP), and Employment Insurance (EI); can also include union dues, and health or dental insurance
- **diameter –** the distance across the centre of a circle; the abbreviation is *d*



- **dimensions –** measurements such as the length, width, or height of an object

E

- **exchange rate –** a rate that specifies how much one currency is worth in terms of another; also known as the foreign-exchange rate

F

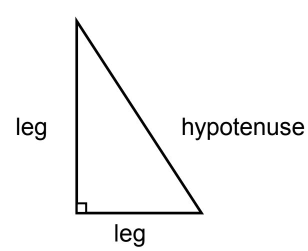
- **Fahrenheit –** a scale for measuring temperature in which the freezing point of water is 32^0F and the boiling point is 212^0F; abbreviation is ^{0}F
- **fluid ounce –** imperial unit of measure for capacity; abbreviation is fl oz; $128 \text{ fl oz} = 1 \text{ gal}$
- **foot –** the basic unit of length in the imperial system; the abbreviation is ft or ′

G

- **gallon –** imperial unit of measure for capacity; abbreviation is gal; $1 \text{ gal} = 128 \text{ fl oz}$
- **gross pay –** total earnings, from a salary, hourly wage, or other payment method, such as tips

H

- **hourly wage –** the amount of money paid to a worker per hour worked
- **hypotenuse –** the longest side of a right triangle; it is located opposite the right angle



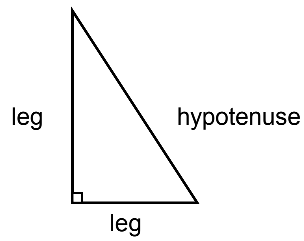
I

- **imperial system –** the system of measurement based on British units

- **inch –** a unit of length in the imperial system; 12 in = 1 ft; the abbreviation is in or ″

L

- **leg –** one of the two shorter sides that forms the right angle in a right triangle

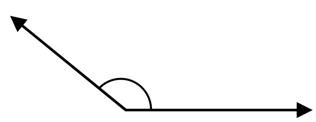


N

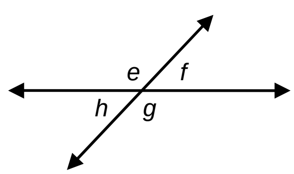
- **net pay –** the amount you receive on a paycheque after deductions have been taken off; also called take-home pay

O

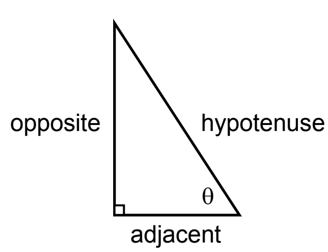
- **obtuse angle –** measures between 90^0 and 180^0



- **opposite angles –** a pair of equal angles formed by two lines that cross; they form an X pattern; in the diagram below, $e = g$ and $f = h$



- **opposite side –** the side across from the angle in a right triangle that you are working with

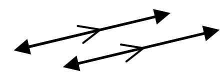


- **ounce –** imperial unit of measure for weight; abbreviation is oz; 16 oz = 1 lb

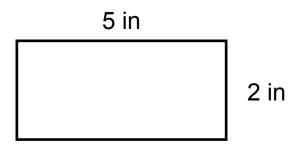
- **overtime –** payment for work done in addition to regular hours; overtime is usually equal to 1.5 times regular pay, but it can be more

P

- **parallel lines –** lines that do not cross each other; are marked by matching arrowheads; two parallel lines are always the same distance apart

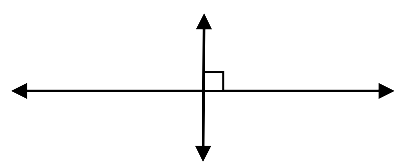


- **perimeter –** the distance around the outside of an object; the symbol for perimeter is *P*



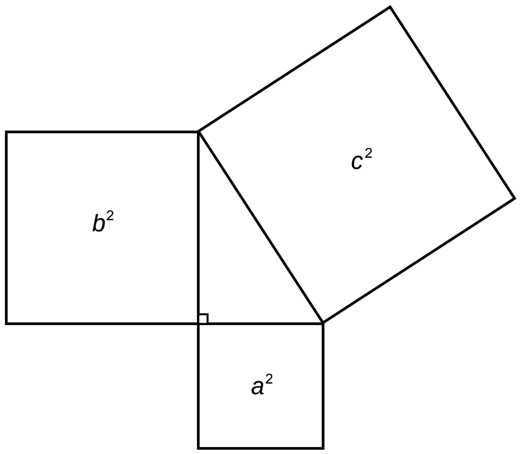
$$P = 2 + 5 + 2 + 5 = 14 \text{ in}$$

- **perpendicular lines –** lines that cross each other at right angles; often marked with one right angle symbol



- **piecework –** an amount of money earned per unit of work; for example, a unit of work may be a planted tree, a completed piece of clothing, or a typed article
- **polygon –** a two-dimensional closed figure with edges that are line segments
- **pound –** imperial unit of measure for weight; abbreviation is lb; 1 lb = 16 oz
- **primary trigonometric ratios –** the three ratios sine, cosine, and tangent, defined in a right triangle
- **proportion –** an equation that says two rates or ratios are equal; for example, $\frac{1}{4} = \frac{4}{16}$

- **Pythagorean relationship –** the relationship among the lengths of the sides of a right triangle; the sum of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse



$a^2 + b^2 = c^2$

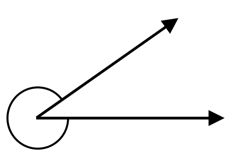
Q

- **quart –** imperial unit of measure for capacity; abbreviation is qt; $1\text{ qt} = 4\text{ cups}$

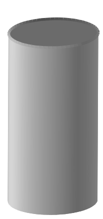
R

- **rectangular prism –** a 3-D object in which the six sides are made up of three pairs of rectangles

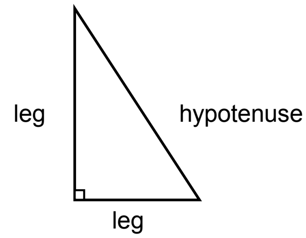
- **reflex angle –** measures more than 180^0 but less than 360^0



- **right cylinder –** a 3-D object with two parallel circular bases

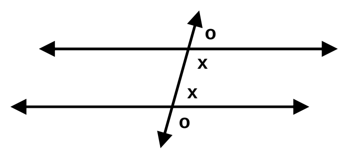


- **right triangle –** a triangle with an angle of 90^0

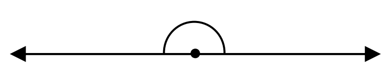


S

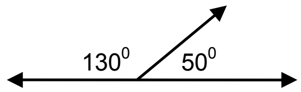
- **salary –** a fixed amount of money paid to a person on a monthly or annual basis; does not depend on the number of hours worked
- **same side exterior angles –** a pair of angles on the same side of a transversal and outside the parallel lines; they add up to 180^0; denoted by **o** in the diagram below



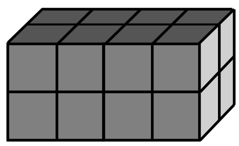
- **same side interior angles –** a pair of angles on the same side of a transversal and inside the parallel lines; they add up to 180^0; they form a C pattern; denoted by **x** in the diagram above
- **shift premium –** an additional payment made for working undesirable shifts, such as overnight shifts
- **SI (Système international d'unités) –** a system of measurement in which units are based on powers of 10; also called the metric system of measurement
- **similar figures –** have the same shape but different size; have equal corresponding angles; have proportional corresponding sides
- **sine ratio –** for an angle in a right triangle, the ratio of the length of the opposite side to the length of the hypotenuse; the short form for the sine ratio of $\angle A$ is $\sin A$; $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$
- **square** – a corner that is exactly 90^0
- **straight angle –** measures exactly 180^0



- **supplementary angles –** two angles that add up to 180^0; they form a straight angle



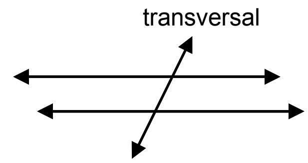
- **surface area –** the number of square units needed to cover an object



T

- **tangent ratio –** for an angle in a right triangle, the ratio of the length of the opposite side to the length of the adjacent side; the short form for the tangent ratio of $\angle A$ is $\tan A$; $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

- **transversal –** a line that crosses two or more parallel lines



U

- **unit price –** the price for one unit of an item; examples include \$2.25/litre, \$5.90/metre, 50¢/apple

Y

- **yard –** a unit of length in the imperial system; $1 \text{ yd} = 3 \text{ ft}$; the abbreviation is yd

SOLUTIONS TO POSSIBLE ASSESSMENT STRATEGIES

SECTION 1.1

- a. 1¢/sheet
  b. 67¢/bottle
- $4.90
- 10 pencils for $4.00 → 40¢/pencil
  6 pencils for $2.70 → 45¢/pencil
  10 pencils for $4.00 is the better buy
- 6 washes for $33 → $5.50/wash
  2 washes for $11.50 → $5.75/wash
  1 wash for $5.95 → $5.95/wash
  6 washes for $33 is the least expensive
- Discount: $2.70; Cost: $15.30
- Discount: $8; Percent: 40%
- $44
- $8/h

SECTION 1.2

- US$502.20
- C$698.60

SECTION 1.3

- 59^0F
- 37.8^0C
- -40^0
- 2.3 kg
- 26.5 lb
- 170 g
- 4.2 oz
- 6.6 gal
- 136.3 L
- 14 qt
- $3.77

SECTION 2.1

- a. 3 ft 2 in
  b. 3 ft 3 in
- a. 414 yd
  b. 39 in
  c. $1\frac{3}{4}$ ft
- 60 installations
- No, because the sum of the length, width, and height is 52 in.

SECTION 2.2

- a. 6 cm = 60 mm
  b. 12 m = 1200 cm
  c. 3.7 dm = 0.37 m
  d. 1.34 m = 1340 mm
  e. 7000 m = 7 km
- 36 cm

SECTION 2.3

- 12.7 cm
- 5.9 in
- 8.0 km
- 19.9 mi
- 2.5 m is approximately equivalent to 8 ft 2.4 in, so it will fit under the bridge
- 5 ft 4 in
- 44 mi

SECTION 2.4

- a. 25.1 cm
  b. 50.3 in
- 130 m
- a. 3.5 in
  b. 41 cm
  c. $4\frac{1}{8}$ m
  d. $\frac{7}{8}$ ft

SECTION 3.1

- a. $3 \text{ ft}^2 = 432 \text{ in}^2$
  b. $\frac{1}{4} \text{ ft}^2 = 36 \text{ in}^2$
  c. $576 \text{ in}^2 = 4 \text{ ft}^2$
  d. $18 \text{ ft}^2 = 2 \text{ yd}^2$
- 3 cans
- 50.3 in^2
- 7.2 ft^2

SECTION 3.2

- a. $2\ m^2 = 20{,}000\ cm^2$

  b. $0.4\ m^2 = 4000\ cm^2$

  c. $800\ mm^2 = 8\ cm^2$

  d. $5{,}000{,}000\ m^2 = 5\ km^2$
- 162.9 cm^2
- 1.62 m^2
- 4000 bricks

SECTION 3.3

- a. 225 m^2

  b. 397.1 ft^2

  c. 308 cm^2
- The area of the larger rectangle is 4 times larger than the smaller rectangle.

SECTION 3.4

- 62 ft^2
- 6 cm
- 326.7 cm^2
- Can A: 45.2 in^2; Can B: 42.4 in^2

  Can A requires more aluminum to manufacture.

SECTION 4.1

- $235
- $11.75
- $1500
- $374
- a. $18.75

  b. $28.35

  c. $15.00
- $400.20
- $1070

SECTION 4.2

- a. $24.88

  b. $9.86
- $694.69
- $676.81

SECTION 4.3

- $372.50
- $2476
- $66,080
- $31.34
- $753.75

SECTION 5.1

- a. $\frac{1}{8}$

  b. $\frac{1}{3}$

  c. $\frac{3}{4}$
- a. obtuse

  b. acute

  c. reflex

  d. right

  e. straight

SECTION 5.2

- a. 58^0

  b. 39^0

  c. 17.5^0

SECTION 5.3

- a. 34^0

  b. 58^0

  c. 8^0
- a. 54^0

  b. 142^0

  c. 90^0
- a. 60^0

  b. 60^0

  c. 120^0

  d. 120^0

  e, 60^0

  f, 60^0

  g. 120^0

SECTION 5.4

- 120^0
- 65^0

SECTION 6.1

- Any answer between 50 m and 80 m is acceptable. The calculated answer is 58.3 m, correct to one decimal place.

SECTION 6.2

- 7 cm and 12 cm
- a. 15.3 cm
  b. 12.3 m
- Squaring the length of each side gives 49, 121, and 225. Since $49 + 121 \neq 225$, the Pythagorean relationship does not hold, therefore the triangle is not right.
- 21 in

SECTION 6.3

- a. yes
  b. no
  c. yes
- Squaring the length of each side and the diagonal gives 576, 324, and 900. Since $576 + 324 = 900$, the Pythagorean relationship holds, therefore the triangle formed by the sides and the diagonal is right, and the picture frame is square.

SECTION 7.1

- 22.5 cm by 31.5 cm
- 5.4 in
- 19.7 m
- 7 m
- Each side of the large triangle is equal to 1.8 times the length of the corresponding side of the small triangle, so the triangles are similar.

SECTION 7.2

- a. $\frac{5}{12}$
  b. $\frac{12}{5}$
- a. 1.9626
  b. 0.4040
  c. 1.1504
- 66.8
- 3.9 m

SECTION 7.3

- a. $\frac{5}{13}$
  b. $\frac{12}{13}$
  c. $\frac{12}{13}$
  d. $\frac{5}{13}$
- a. 0.8387
  b. 0.9205
- 5.8
- 25.6 km

SECTION 7.4

- a. 27.1^0
  b. 50.4^0
  c. 26.0^0
  d. 71.5^0
- 43.8^0
- 32.0^0
- 45.1^0
- 65.0^0

APPENDIX

MATHEMATICS RESEARCH PROJECT

Introduction

A research project can be a very important part of a mathematics education. Besides the greatly increased learning intensity that comes from personal involvement with a project, and the chance to show universities, colleges, and potential employers the ability to initiate and carry out a complex task, it gives the student an introduction to mathematics as it is – a living and developing intellectual discipline where progress is achieved by the interplay of individual creativity and collective knowledge. A major research project must successfully pass through several stages. Over the next few pages, these stages are highlighted, together with some ideas that students may find useful when working through such a project.

Creating an Action Plan

As previously mentioned, a major research project must successfully pass though several stages. The following is an outline for an action plan, with a list of these stages, a suggested time, and space for students to include a probable time to complete each stage.

STAGE	SUGGESTED TIME	PROBABLE TIME
Select the topic to explore.	1 – 3 days	
Create the research question to be answered.	1 – 3 days	
Collect the data.	5 – 10 days	
Analyse the data.	5 – 10 days	
Create an outline for the presentation.	2 – 4 days	
Prepare a first draft.	3 – 10 days	
Revise, edit and proofread.	3 – 5 days	
Prepare and practise the presentation.	3 – 5 days	

Completing this action plan will help students organize their time and give them goals and deadlines that they can manage. The times that are suggested for each stage are only a guide. Students can adjust the time that they will spend on each stage to match the scope of their projects. For example, a project based on primary data (data that they collect) will usually require more time than a project based on secondary data (data that other people have collected and published). A student will also need to consider his or her personal situation – the issues that each student deals with that may interfere with the completion of his or her project. Examples of these issues may include:

- a part-time job;
- after-school sports and activities;
- regular homework;
- assignments for other courses;
- tests in other courses;
- time they spend with friends;
- family commitments;
- access to research sources and technology.

➢ Selecting the Research Topic

To decide what to research, a student can start by thinking about a subject and then consider specific topics. Some examples of subjects and topics may be:

SUBJECT	TOPIC
Entertainment	• effects of new electronic devices • file sharing
Health care	• doctor and/or nurse shortages • funding
Post-secondary education	• entry requirements • graduate success
History of Western and Northern Canada	• relations among First Nations • immigration

It is important to take the time to consider several topics carefully before selecting a topic for a research project. The following questions will help a student determine if a topic that is being considered is suitable.

- **Does the topic interest the student?**
  Students will be more successful they choose a topic that interests them. They will be more motivated to do research, and they will be more attentive while doing the research. As well, they will care more about the conclusions they make.

- **Is the topic practical to research?**
  If a student decides to use first-hand data, can the data be generated in the time available, with the resources available? If a student decides to use second-hand data, are there multiple sources of data? Are the sources reliable, and can they be accessed in a timely manner?

- **Is there an important issue related to the topic?**
  A student should think about the issues related to the topic that he or she has chosen. If there are many viewpoints on an issue, they may be able to gather data that support some viewpoints but not others. The data they collect from all viewpoints should enable them to come to a reasoned conclusion.

- **Will the audience appreciate the presentation?**
  The topic should be interesting to the intended audience. Students should avoid topics that may offend some members of their audience.

➢ Creating the Research Question or Statement

A well-written research question or statement clarifies exactly what the project is designed to do. It should have the following characteristics:

- The research topic is easily identifiable.

- The purpose of the research is clear.
- The question or statement is focused. The people who are listening to or reading the question or statement will know what the student is going to be researching.

A good question or statement requires thought and planning. Below are three examples of initial questions or statements and how they were improved.

UNACCEPTABLE QUESTION OR STATEMENT	WHY?	ACCEPTABLE QUESTION OR STATEMENT
Is mathematics used in computer technology?	Too general	What role has mathematics played in the development of computer animation?
Water is a shared resource.	Too general	Homes, farms, ranches, and businesses east of the Rockies all use runoff water. When there is a shortage, that water must be shared.
Do driver’s education programs help teenagers parallel park?	Too specific, unless the student is generating his or her own data	Do driver’s education programs reduce the incidence of parking accidents?

The following checklist can be used to determine if the research question or statement is effective.

- Does the question or statement clearly identify the main objective of the research? After the question or statement is read to someone, can they tell what the student will be researching?
- Is the student confident that the question or statement will lead him or her to sufficient data to reach a conclusion?
- Is the question or statement interesting? Does it make the student want to learn more?
- Is the topic that the student chose purely factual, or is that student likely to encounter an issue, with different points of view?

➢ Carrying Out the Research

As students continue with their projects, they will need to conduct research and collect data. The strategies that follow will help them in their data collection.

There are two types of data that students will need to consider – primary and secondary. Primary data is data that the student collects himself or herself using surveys, interviews and direct observations. Secondary data is data that the student obtains through other sources, such as online publications, journals, magazines, and newspapers.

Both primary and secondary data have their advantages and disadvantages. Primary data provide specific information about the research question or statement, but may take time to collect and process. Secondary data is usually easier to obtain and can be analysed in less time. However, because the data was originally gathered for other purposes, a student may need to sift through it to find exactly what he or she is looking for.

The type of data chosen can depend on many factors, including the research question, the skills of the student, and available time and resources. Based on these and other factors, the student may chose to use primary data, secondary data, or both.

When collecting primary data, the student must ensure the following:

- For surveys, the sample size must be reasonably large and the random sampling technique must be well designed.
- For surveys and interviews, the questionnaires must be designed to avoid bias.
- For experiments and studies, the data must be free from measurement bias.
- The data must be compiled accurately.

When collecting secondary data, the student should explore a variety of resources, such as:

- textbooks, and other reference books;
- scientific and historical journals, and other expert publications;
- the Internet;
- library databases.

After collecting the secondary data, the student must ensure that the source of the data is reliable:

- If the data is from a report, determine what the author's credentials are, how recent the data is, and whether other researchers have cited the same data.
- Be aware that data collection is often funded by an organization with an interest in the outcome or with an agenda that it is trying to further. Knowing which organization has funded the data collection may help the student decide how reliable the data is, or what type of bias may have influenced the collection or presentation of the data.
- If the data is from the Internet, check it against the following criteria:
    - authority – the credentials of the author should be provided;
    - accuracy – the domain of the web address may help the student determine the accuracy;
    - currency – the information is probably being accurately managed if pages on a site are updated regularly and links are valid.

➢ Analysing the Data

Statistical tools can help a student analyse and interpret the data that is collected. Students need to think carefully about which statistical tools to use and when to use them, because other people will be scrutinizing the data. A summary of relevant tools follows.

Measures of central tendency will give information about which values are representative of the entire set of data. Selecting which measure of central tendency (mean, median, or mode) to use depends on the distribution of the data. As the researcher, the student must decide which measure most accurately describes the tendencies of the population. The following criteria should be considered when deciding upon which measure of central tendency best describes a set of data.

- Outliers affect the mean the most. If the data includes outliers, the student should use the median to avoid misrepresenting the data. If the student chooses to use the mean, the outliers should be removed before calculating the mean.

- If the distribution of the data is not symmetrical, but instead strongly skewed, the median may best represent the set of data.
- If the distribution of the data is roughly symmetrical, the mean and median will be close, so either may be appropriate to use.
- If the data is not numeric (for example, colour), or if the frequency of the data is more important than the values, use the mode.

Both the range and the standard deviation will give the student information about the distribution of data in a set. The range of a set of data changes considerably because of outliers. The disadvantage of using the range is that it does not show where most of the data in a set lies – it only shows the spread between the highest and the lowest values. The range is an informative tool that can be used to supplement other measures, such as standard deviation, but it is rarely used as the only measure of dispersion.

Standard deviation is the measure of dispersion that is most commonly used in statistical analysis when the mean is used to calculate central tendency. It measures the spread relative to the mean for most of the data in the set. Outliers can affect standard deviation significantly. Standard deviation is a very useful measure of dispersion for symmetrical distributions with no outliers. Standard deviation helps with comparing the spread of two sets of data that have approximately the same mean. For example, the set of data with the smaller standard deviation has a narrower spread of measurement around the mean, and therefore has comparatively fewer high or low scores, than a set of data with a higher standard deviation.

When working with several sets of data that approximate normal distributions, you can use *z*-scores to compare the data values. A *z*-score table enables a student to find the area under a normal distribution curve with a mean of zero and a standard deviation of one. To determine the *z*-score for any data value in a set that is normally distributed, the formula $z = \frac{x - \overline{x}}{s}$ can be used where x is any observed data value in the set, $\overline{x}$ is the mean of the set, and is s is the standard deviation of the set.

When analysing the results of a survey, a student may need to interpret and explain the significance of some additional statistics. Most surveys and polls draw their conclusions from a sample of a larger group. The margin of error and the confidence level indicate how well a sample represents a larger group. For example, a survey may have a margin of error of plus or minus 3% at a 95% level of confidence. This means that if the survey were conducted 100 times, the data would be within 3 percent points above or below the reported results in 95 of the 100 surveys.

The size of the sample that is used for a poll affects the margin of error. If a student is collecting data, he or she must consider the size of the sample that is needed for a desired margin of error.

➢ Identifying Controversial Issues

While working on a research project, a student may uncover some issues on which people disagree. To decide on how to present an issue fairly, he or she should consider some questions to ask as the research proceeds.

- **What is the issue about?**

  The student should identify which type of controversy has been uncovered. Almost all controversy revolves around one or more of the following:
  - Values – What should be? What is best?
  - Information – What is the truth? What is a reasonable interpretation?

  - o Concepts – What does this mean? What are the implications?

- **What positions are being taken on the issue?**

  The student should determine what is being said and whether there is reasonable support for the claims being made. Some questions to ask are:
  - o Would you like that done to you?
  - o Is the claim based on a value that is generally shared?
  - o Is there adequate information?
  - o Are the claims in the information accurate?
  - o Are those taking various positions on the issue all using the same term definitions?

- **What is being assumed?**

  Faulty assumptions reduce legitimacy. The student can ask:
  - o What are the assumptions behind an argument?
  - o Is the position based on prejudice or an attitude contrary to universally held human values, such as those set out in the United Nations Declaration of Human Rights?
  - o Is the person who is presenting a position or an opinion an insider or an outsider?

- **What are the interests of those taking positions?**

  The student should try to determine the motivations of those taking positions on the issue. What are their reasons for taking their positions? The degree to which the parties involved are acting in self-interest could affect the legitimacy of their opinions.

➢ The Final Product and Presentation

The final presentation should be more than just a factual written report of the information gathered from the research. To make the most of the student's hard work, he or she should select a format for the final presentation that suits his or her strengths, as well as the topic.

To make the presentation interesting, a format should be chosen that suits the student's style. Some examples are:

- a report on an experiment or an investigation;
- a summary of a newspaper article or a case study;
- a short story, musical performance, or play;
- a web page;
- a slide show, multimedia presentation, or video;
- a debate;
- an advertising campaign or pamphlet;
- a demonstration or the teaching of a lesson.

Sometimes, it is also effective to give the audience an executive summary of the presentation. This is a one-page summary of the presentation that includes the research question and the conclusions that were made.

Before giving the presentation, the student can use these questions to decide if the presentation will be effective.

- Did I define my topic well? What is the best way to define my topic?
- Is my presentation focused? Will my classmates find it focused?
- Did I organize my information effectively? Is it obvious that I am following a plan in my presentation?
- Am I satisfied with my presentation? What might make it more effective?
- What unanswered questions might my audience have?

➢ Peer Critiquing of Research Projects

After the student has completed his or her research for the question or statement being studied, and the report and presentation have been delivered, it is time to see and hear the research projects developed by other students. However, rather than being a passive observer, the student should have an important role – to provide feedback to his or her peers about their projects and presentations.

Critiquing a project does not involve commenting on what might have been or should have been. It involves reacting to what is seen and heard. In particular, the student should pay attention to:

- strengths and weaknesses of the presentation;
- problems or concerns with the presentation.

While observing each presentation, students should consider the content, the organization, and the delivery. They should take notes during the presentation, using the following rating scales as a guide. These rating scales can also be used to assess the presentation.

Content

Shows a clear sense of audience and purpose.	1	2	3	4	5
Demonstrates a thorough understanding of the topic.	1	2	3	4	5
Clearly and concisely explains ideas.	1	2	3	4	5
Applies knowledge and skills developed in this course.	1	2	3	4	5
Justifies conclusions with sound reasoning.	1	2	3	4	5
Uses vocabulary, symbols and diagrams correctly.	1	2	3	4	5

Organization

Presentation is clearly focused.	1	2	3	4	5
Engaging introduction includes the research question, clearly stated.	1	2	3	4	5
Key ideas and information are logically presented.	1	2	3	4	5
There are effective transitions between ideas and information.	1	2	3	4	5
Conclusion follows logically from the analysis and relates to the question.	1	2	3	4	5

Delivery

Speaking voice is clear, relaxed, and audible.	1	2	3	4	5
Pacing is appropriate and effective for the allotted time.	1	2	3	4	5
Technology is used effectively.	1	2	3	4	5
Visuals and handouts are easily understood.	1	2	3	4	5
Responses to audience’s questions show a thorough understanding of the topic.	1	2	3	4	5

REFERENCES

American Association for the Advancement of Science [AAAS-Benchmarks]. *Benchmark for Science Literacy*. New York, NY: Oxford University Press, 1993.

Banks, James A. and Cherry A. McGee Banks. *Multicultural Education: Issues and Perspectives*. Boston: Allyn and Bacon, 1993.

Black, Paul and Dylan Wiliam. “Inside the Black Box: Raising Standards Through Classroom Assessment.” *Phi Delta Kappan*, 20, October 1998, pp.139-148.

Canavan-McGrath, Cathy, *et. al. Foundations of Mathematics 11*. Nelson Education, 2012.

British Columbia Ministry of Education. *The Primary Program: A Framework for Teaching*, 2000.

Davies, Anne. *Making Classroom Assessment Work*. British Columbia: Classroom Connections International, Inc., 2000.

Hope, Jack A., *et. al*. *Mental Math in the Primary Grades*. Dale Seymour Publications, 1988.

National Council of Teachers of Mathematics. *Mathematics Assessment: A Practical Handbook*. Reston, VA: NCTM, 2001.

National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

Rubenstein, Rheta N. *Mental Mathematics Beyond the Middle School: Why? What? How?* September 2001, Vol. 94, Issue 6, p. 442.

Shaw, Jean M. and Mary Jo Puckett Cliatt. “Developing Measurement Sense.” In P.R. Trafton (ed.), *New Directions for Elementary School Mathematics* (pp. 149–155). Reston, VA: NCTM, 1989.

Steen, Lynn Arthur (ed.). *On the Shoulders of Giants – New Approaches to Numeracy.* Washington, DC: National Research Council, 1990.

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades 5-8.* Boston: Pearson Education, Inc. 2006.

Western and Northern Canadian Protocol. *Common Curriculum Framework for 10-12 Mathematics*, 2008.